Masking Countermeasures in Cryptographic Hardware: Part I

Cryptographic Hardware for Embedded Systems

ECE 3170

Fall 2025

Assoc. Prof. Vincent John Mooney III

Georgia Institute of Technology

Reading

- This lecture is based on three sources:
- Chapter 9 of *Power Analysis Attacks: Revealing the Secrets of Smart Cards* by Mangard et al., 2007, ISBN-13: 978-0-387-30857-9, ISBN-10: 0-387-30857-1, e-ISBN-10: 0-387-38162-7.
- L. Goubin and J. Patarin, "DES and Differential Power Analysis The 'Duplication' Method," Cryptographic Hardware for Embedded Systems (CHES) conference, 1999.
- Chapter 2 of *Handbook of Applied Cryptography* by Menezes et al., 1996, ISBN: 978-1-119-09672-6.

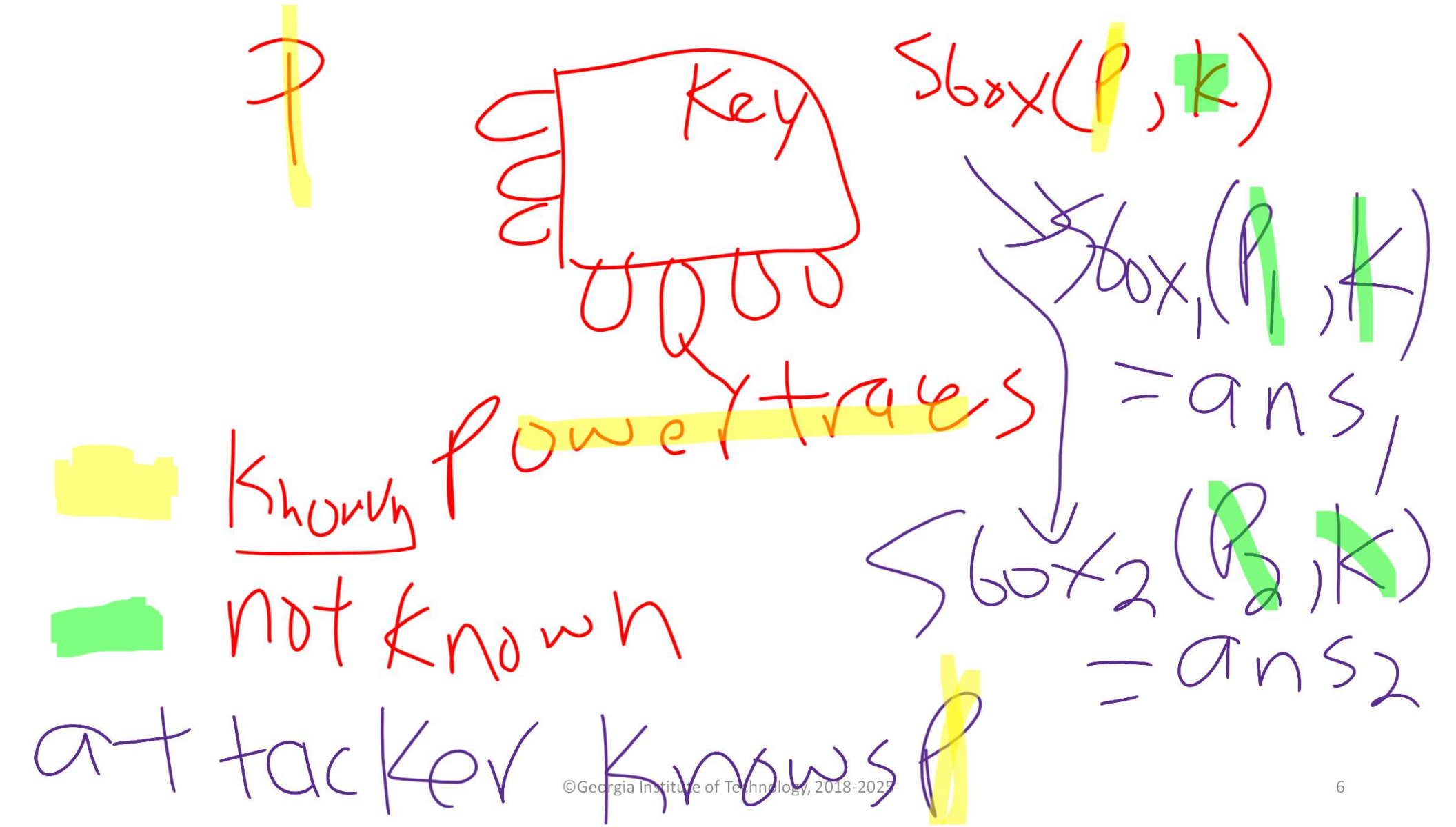
Mathematical Background

- Recall that $\mathbb{Z}$ is the set of integers (including negative numbers and zero)
  - The CHES paper by Goubin and Patarin use **Z** instead of $\mathbb{Z}$
- Let n be a positive integer. Then $\mathbb{Z}_n$ is {0,1,2,…,n-1}
- gcd(x,y) is the greatest common divisor of x and y
- The multiplicative group of $\mathbb{Z}_n$ is $\mathbb{Z}_n^*$ = {a ϵ $\mathbb{Z}_n$ | gcd(a,n) = 1}. In particular, if n is prime, then $\mathbb{Z}_n^*$ = {a | 1 ≤ a ≤ n-1}.
- Recall that $\oplus$ is linear, i.e., $f(V_1 \oplus V_2) = f(V_1) \oplus f(V_2)$
- S-boxes are nonlinear, i.e., $S(V_1 \oplus V_2) \neq S(V_1) \oplus S(V_2)$

Main Idea

- Replace each intermediate variable V with k variables $V_1,\ldots,V_k$ such that $V_1,\ldots,V_k$ can be used to recover (calculate) V
- Condition 1: from the knowledge of V and for any i (where $1 \le i \le k$), it is not feasible to deduce information about the set of possible values of V_i such that there exists values $V_1,\ldots,V_{i-1},V_{i+1},\ldots,V_k$ satisfying the equation $f(V_1,\ldots,V_k) = V$
  - Obviously, take for example V_i has 8 bits, clearly V_i is equal to an 8-bit value between 0x00 and 0xFF. This fact is not information "deduced" about V_i from the value of V
- Condition 2: the function f() is such that the transformations to be performed on V_1, V_2, …, or V_k during the computation (instead of transformations performed on V) can be implemented without explicit calculation of V

reg file is not attacked, no leakage



Example of Condition 1

- Choose f(V_1,…,V_k) = $V_1 \oplus V_2 \oplus \ldots \oplus V_k$
- Clearly, for any particular i (where $1 \le i \le k$), V_i can take on any value
  - Therefore, even with knowledge of V, no limitation is placed on the value of V_i

Example of Condition 2

- Let $V \in$ multiplicative group $\mathbb{Z}_n^*$
  - The CHES paper by Goubin and Patarin use $\mathbb{Z}/n\mathbb{Z}$ instead of $\mathbb{Z}_n^*$ to indicate a multiplicative group
- $f(V_1,\ldots,V_k) = V_1 * V_2 * \ldots * V_k \bmod n$
  - where, for each i, $1 \le i \le k$, $V_i \in$ multiplicative group $\mathbb{Z}_n^*$
- Clearly, for $f(V_1,\ldots,V_k)$ as defined, individual transformations can be performed on $V_1, V_2, \ldots, V_k$ without calculating V
- Condition 1 is also satisfied as well

Mathematical Background (cont'd)

- Handbook of Applied Cryptography, Chapter 2.4, pp. 63-75
- Definition
  - The *multiplicative group* of $\mathbb{Z}_n$ is $\mathbb{Z}_n^* = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}$
  - In particular, if *n* is prime, then $\mathbb{Z}_n^* = \{a \mid 1 \leq a \leq n-1\}$
- Definition
  - The *order* of $\mathbb{Z}_n^*$ is the number of elements in $\mathbb{Z}_n^*$, i.e., $|\mathbb{Z}_n^*|$
  - Note that if $a \in \mathbb{Z}_n^*$ and $b \in \mathbb{Z}_n^*$ then $a \cdot b \in \mathbb{Z}_n^*$, i.e., $\mathbb{Z}_n^*$ is closed under multiplication (recall that all multiplication in $\mathbb{Z}_n$ is mod n)
- Example 1: $\mathbb{Z}_{21}^* = \{1,2,4,5,8,10,11,13,16,17,19,20\}$
- Example 2: $\mathbb{Z}_{13}^* = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

Example of Example of Condition 2

- First note the multiplicative groups are important for asymmetric encryption schemes such as RSA
- Consider $\mathbb{Z}_{13}^*$
- 12 = 3 * 4
- So if V = 12, V_1 = 3 and V_2 = 4, f(V_1,…,V_k) = V_1*V_2 mod 13
- The mod function provides the result that Condition 2 holds

A DES Round

- Key bits shifted, then 48 bits selected

1) R_{i-1} expanded to 48 bits
2) Key bits permuted and XORed with R_{i-1}
3) Eight S-boxes produce 32 bits
4) 32 bits are permuted

- Function ***f*** is comprised of the above four steps
- Output of ***f*** XORed w/ L_{i-1}
  - Result: R_i
- $L_i = R_{i-1}$

Recall Slide 7 of Lecture 4 DES !

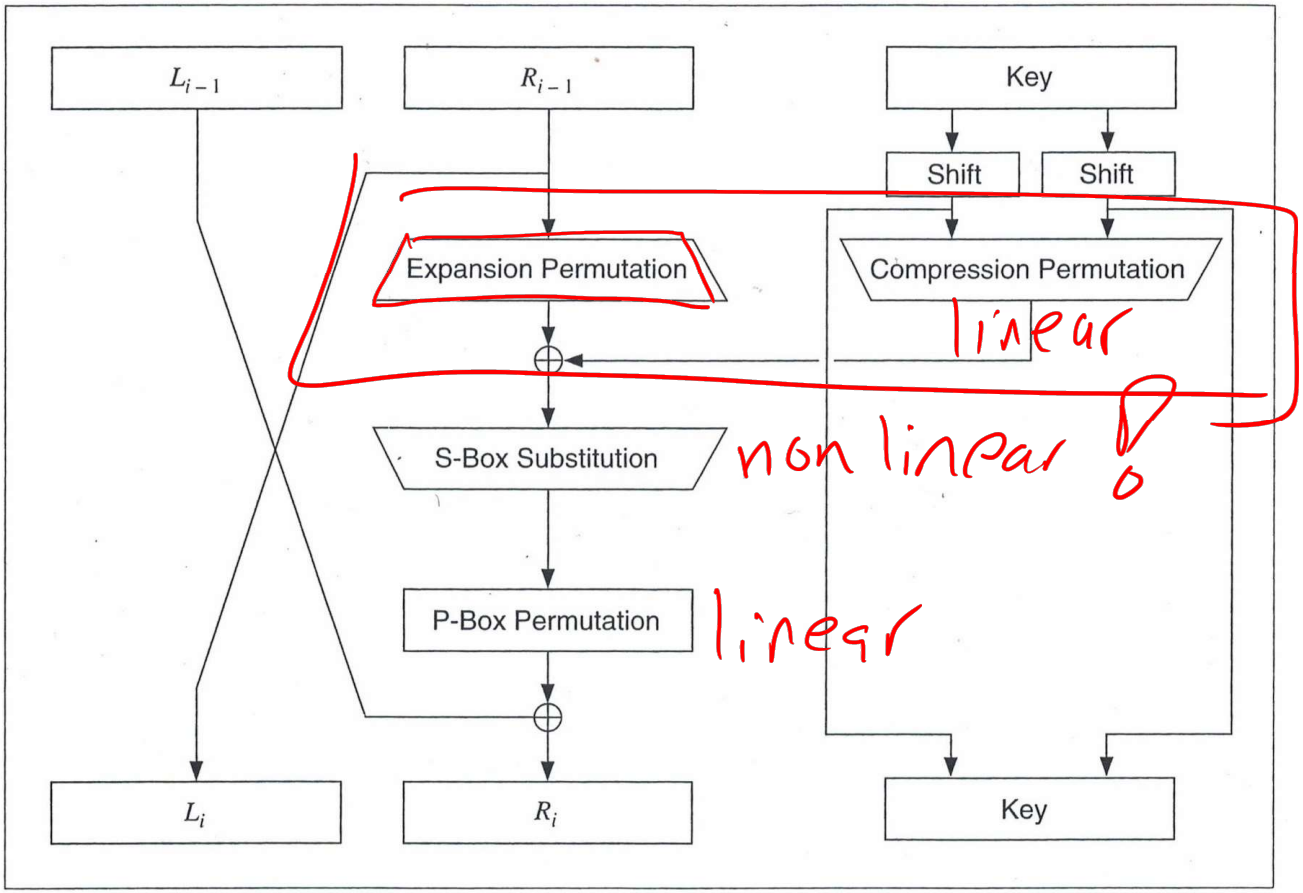


Figure 12.2 One round of DES.

Example: DES

- Consider intermediate variable V
- Separate V into two components: V_1 and V_2
- E.g., choose a function $f(V_1, V_2) = V = V_1 \oplus V_2$
- Condition 1 is satisfied
- All DES transformations fall into one of the following 5 categories:
  - Permutation of the bits of V
  - Expansion of the bits of V
  - $\oplus$ between V and another variable V' of the same type
  - $\oplus$ between V and another variable C depending only on the key
  - Transformations of V using a substitution box

Example: DES (cont'd)

- First two consist of linear transformations
  - To satisfy Condition 2, just perform the permutation and expansion first on V_1 then V_2
  - From linearity, $f(V_1,V_2) = V$ holds after these transformations as well
- For the third category, just replace $V''' = V \oplus V'$ by (1) $V_1''' = V_1 \oplus V_1'$ and (2) $V_2''' = V_2 \oplus V_2'$
  - Also from linearity, $f(V_1,V_2) = V$ and $f(V_1',V_2') = V'$ result in $f(V_1''',V_2''') = V'''$
  - Thus, condition 2 also holds for this category
- The fourth category similarly maintains Condition 2, just replace $V \oplus C$ with $V_1 \oplus C$ (or with $V_2 \oplus C$)

1. Permutation of the bits of V
2. Expansion of the bits of V
3. $\oplus$ between V and another variable V' of the same type
4. $\oplus$ between V and another variable depending only on the key
5. Transformations of V using a substitution box

all linear transformations

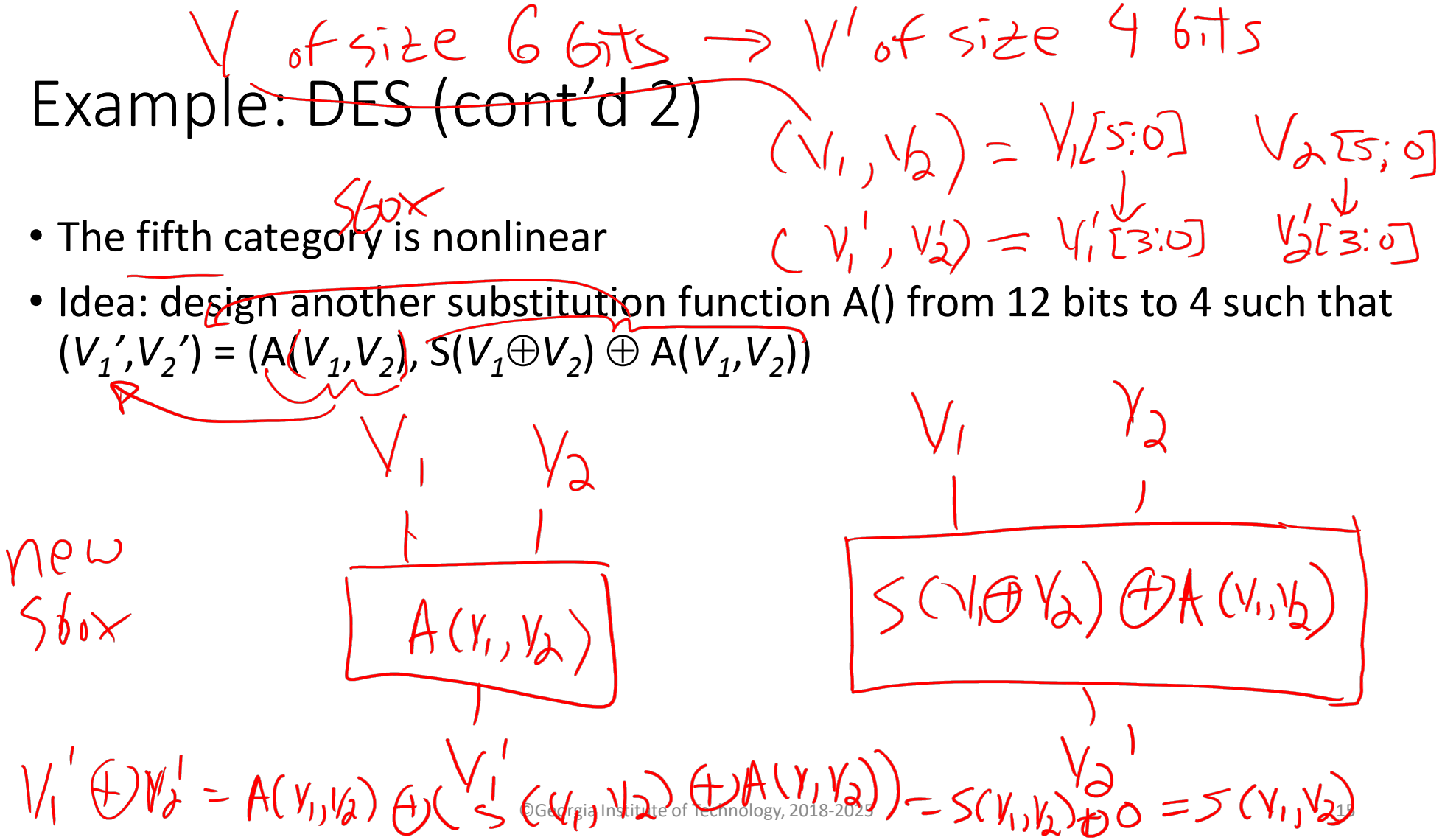
Can be split up into two calc. (and reconciled later)

Main Idea (REPEATED from Slide 4!)

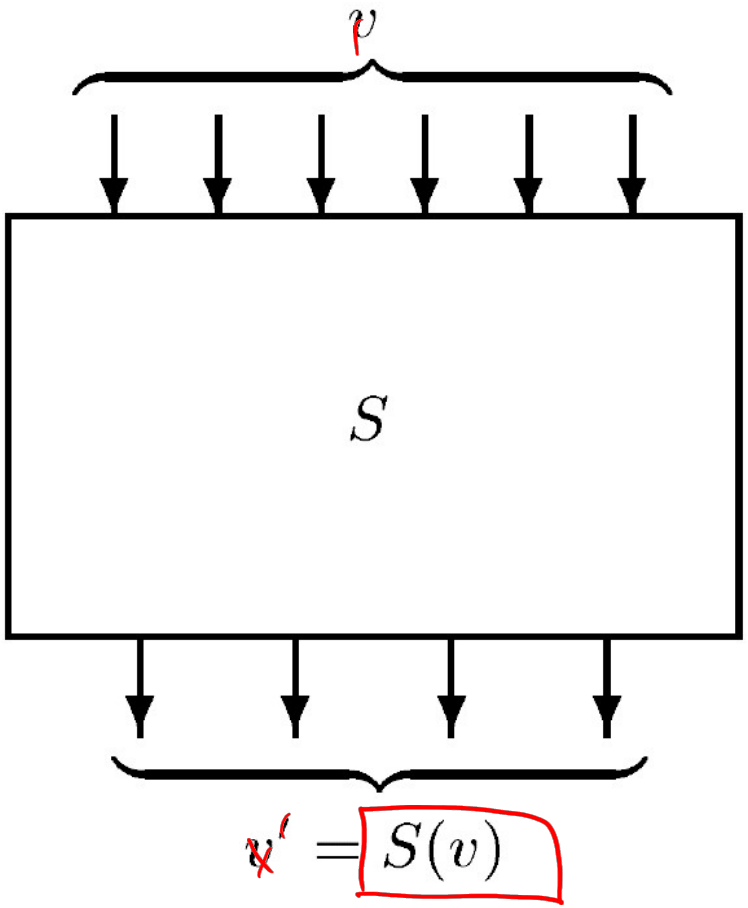
- Replace each intermediate variable V with k variables $V_1,\ldots,V_k$ such that $V_1,\ldots,V_k$ can be used to recover (calculate) V
- Condition 1: from the knowledge of V and for any i (where $1 \le i \le k$), it is not feasible to deduce information about the set of possible values of V_i such that there exists values $V_1,\ldots,V_{i-1},V_{i+1},\ldots,V_k$ satisfying the equation $f(V_1,\ldots,V_k) = V$
  - Obviously, take for example V_i has 8 bits, clearly V_i is equal to an 8-bit value between 0x00 and 0xFF. This fact is not information "deduced" about V_i from the value of V
- Condition 2: the function f() is such that the transformations to be performed on V_1, V_2, …, or V_k during the computation (instead of transformations performed on V) can be implemented without explicit calculation of V

Example: DES (cont'd 2)

- The fifth category is nonlinear
- Idea: design another substitution function A() from 12 bits to 4 such that $(V_1', V_2') = (A(V_1, V_2), S(V_1 \oplus V_2) \oplus A(V_1, V_2))$



base case



6 bits

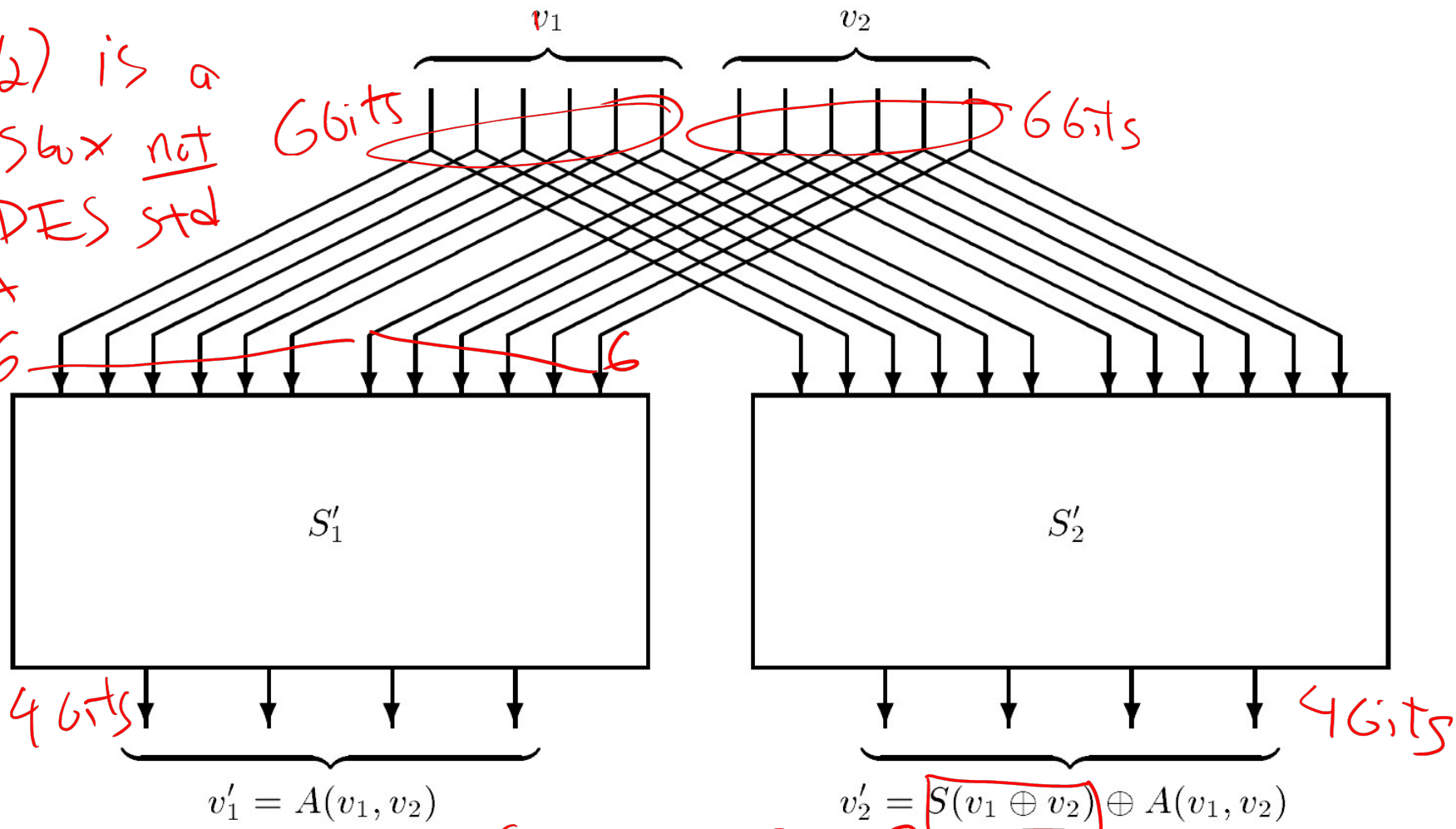
4 bits

Initial implementation: the predictable values

v and v' appear in RAM at some time

A(Y1, Y2) is a new Sbox not in the DES std

Keep A secret

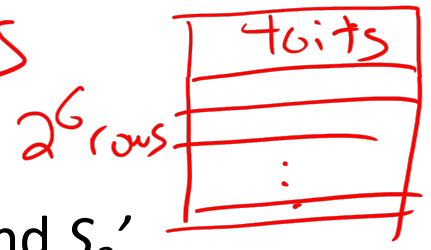


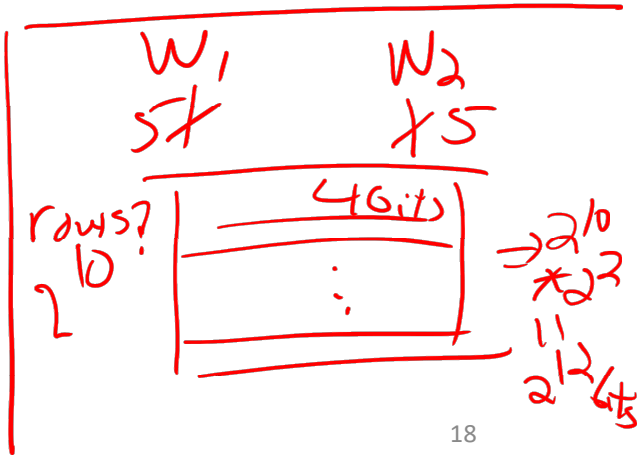
cdc. using only the reg. file, no SRAM!

Modified implementation: the values $v = v_1 \oplus v_2$ and $v' = v'_1 \oplus v'_2$ never explicitely appear in RAM

Example: DES (cont'd 3)

- The result is two larger substitution boxes S_1' and S_2'
- S_1' implements function A from 12 bits to 4 such that $V_1' = A(V_1, V_2)$
- S_2' implements function $S(V_1, V_2) \oplus A(V_1, V_2)$ from 12 bits (V_1, V_2) to 4 (V_2') such that $V_2' = S(V_1 \oplus V_2) \oplus A(V_1, V_2)$
- Substitution function A satisfies Condition 1
- Table look-up never explicitly calculates $V_1 \oplus V_2$
  - Thus, Condition 2 is satisfied





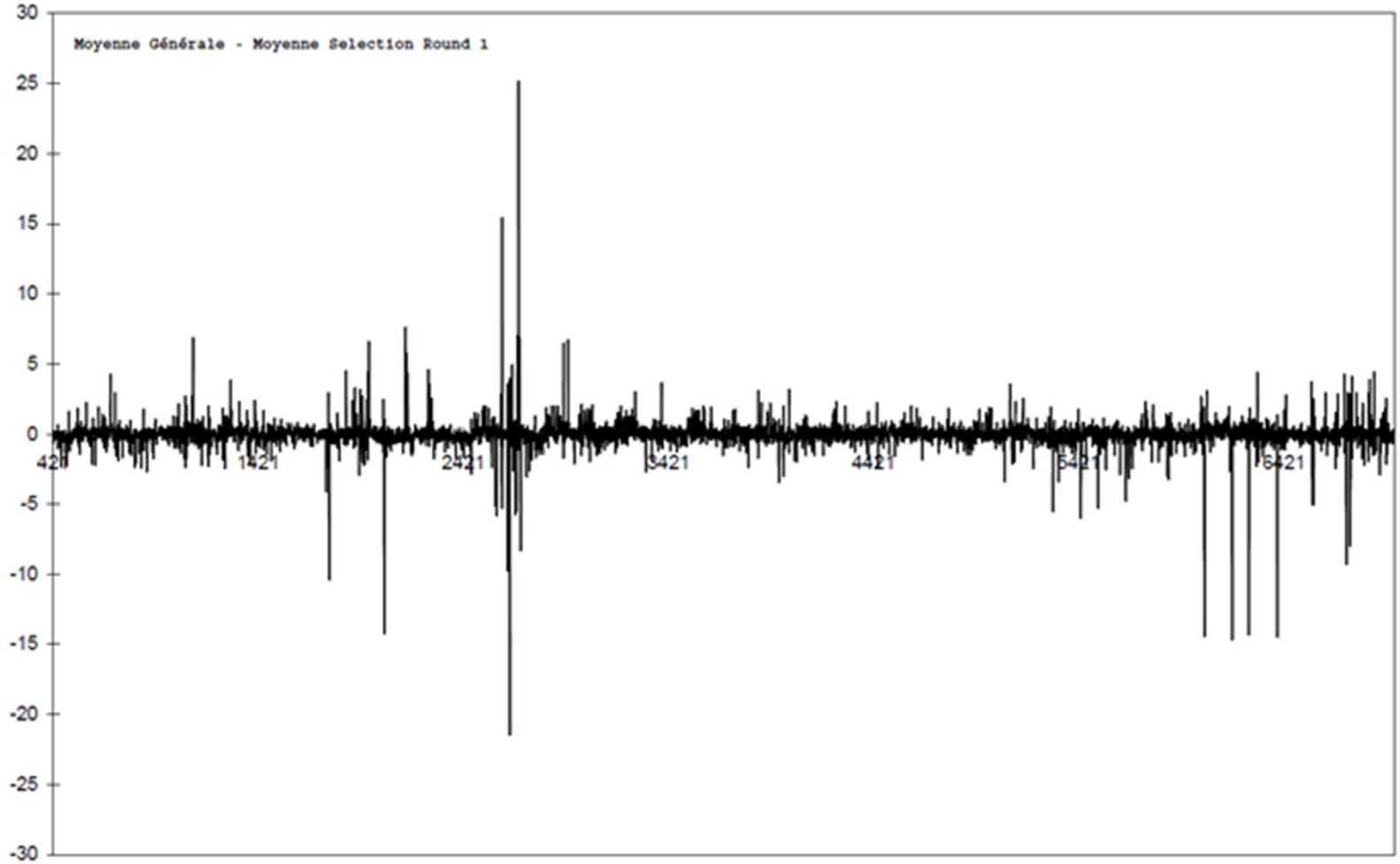


Fig. 6. An example of difference of the curves MC and MC' when the 6 bits are false

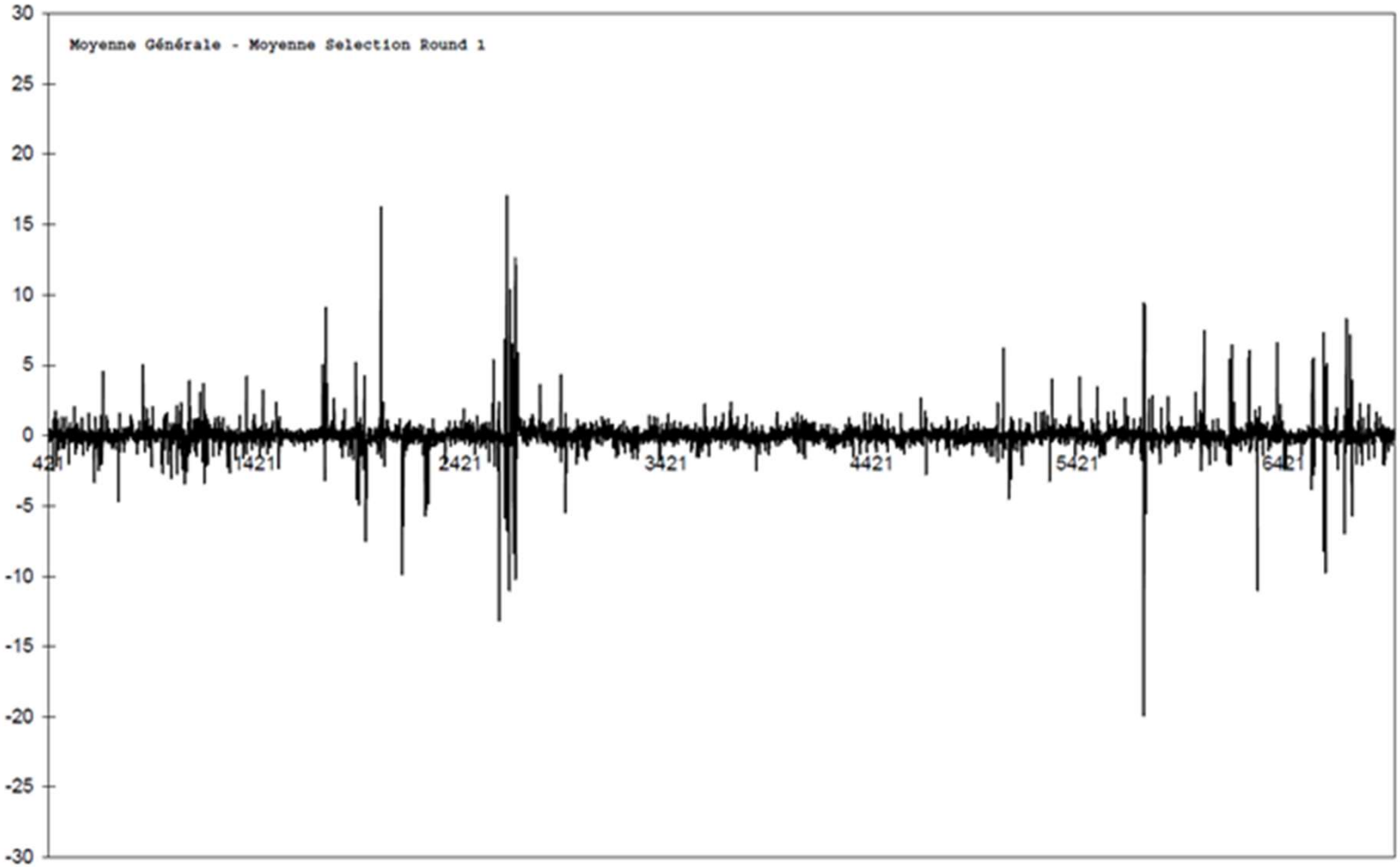


Fig. 7. Difference of the curves MC and MC' when the 6 bits are correct

Summary of Masking Types

- Boolean
  - Intermediate value *v* is concealed by XOR with mask *m*
  - $v_m = v \oplus m$
- Additive
  - Intermediate value *v* is concealed by modular addition with mask *m*
  - $v_m = v + m \pmod{n}$
- Multiplicative
  - Intermediate value *v* is concealed by modular multiplication with mask *m*
  - $v_m = v * m \pmod{n}$

Arithmetic Masking Example: RSA

- Consider the "square and multiply" implementation
  - Perform computation of $x^d \bmod n$
  - Exponent d is part of the key
  - d has m bits $d_{m-1}d_{m-2} \ldots d_1d_0$
    - 1. $z \leftarrow 1$;
    - For i going backwards from m - 1 to 0 do:
    - 2. $z \leftarrow z^2 \bmod n$;
    - 3. if $d_i = 1$ then $z \leftarrow z * x \bmod n$;

$x^d \bmod n = (x_1 * x_2)^d$

$= x_1^d \bmod n * x_2^d \bmod n$

now, calc. x_1^d sep. fr. x_2^d

original alg.

timing side channel

Arithmetic Masking Example: new RSA

- Consider again the "square and multiply" implementation
  - Recall we replace intermediate variable V with variables V_1 and V_2
    - In this case we replace x by x_1 and x_2
    - Need to compute $x_1^d \bmod n$ and $x_2^d \bmod n$
    - Final computation will be $x^d \bmod n = (x_1^d \bmod n)*(x_2^d \bmod n) \bmod n$
  - As before d has m bits $d_{m-1}d_{m-2} \ldots d_1d_0$

1. $z_1 \leftarrow 1$;	1. $z_2 \leftarrow 1$;
For i going backwards from m - 1 to 0 do:	For i going backwards from m - 1 to 0 do:
2. $z_1 \leftarrow z_1^2 \bmod n$;	2. $z_2 \leftarrow z_2^2 \bmod n$;
3. if $d_i = 1$ then $z_1 \leftarrow z_1 * x_1 \bmod n$;	3. if $d_i = 1$ then $z_2 \leftarrow z_2 * x_2 \bmod n$;

  - The above steps can be done in a random fashion including variations on overlapping versus sequential execution