Cryptography Part III: Hash Functions

Cryptographic Hardware for Embedded Systems FCF 3170

Fall 2025

Assoc. Prof. Vincent John Mooney III
Georgia Institute of Technology

Reading Assignment

• Please read chapter 18 of the course textbook by Schneier

One-Way Hash Function Definition by Schneier

- Given a message M of arbitrary length, a hash function H generates a fixedlength output h
 - h = H(M)
- A hash function is one-way if it satisfies the following
 - i. Given M and H, it is easy to compute h
 - ii. Given h and H, it is hard to compute M
 - iii. Given M and H, it is hard to compute M' such that H(M') = H(M)
- A one-way hash function can be used to provide a "signature" of M
 - Note that property iii above makes it hard for an adversary to change the message but not the one-way hash value
 - Property iii above is also generalized as collision resistance

Use Case Scenario

- Message Authentication
 - Recall that in this course we take "message authentication" to mean identity authentication + message integrity (both!)
 - Financial example: bank deposit
 - An adversary should not be able to, for example, alter a message containing a withdrawal (or any other bank transaction, for that matter) and do so undetected
 - In general, hash functions form the basis of message signatures
 - Recall as mentioned in the lecture "Authentication I" that an old-fashioned signature is a known way to provide authentication! We thus need a method of digital signing

MD5 Overview

- Authored by Ronald Rivest, Professor of Electrical Engineering and Computer Science at MIT
 - Co-author of the asymmetric RSA cryptographic algorithm in 1977
 - Invented MD5 in 1991
- Goal: message integrity
- Keyless

MD5 Overview (continued)

- Message is divided into blocks
- MD5 block input size: 512 bits
- MD5 block output size: 128 bits
- Keyless
- Message length: up to 2⁶⁴ blocks

MD5 Input

- The overall message (plaintext) must be a multiple of 512 bits
- The last 512-bit block may only contain 448 bits or less of the plaintext
 - Note that 448 = 512-64
 - The last 64 bits represents the message length
 - If less than 448 bits of plaintext are available for the last block, pad with a one followed by as many zeros as are necessary

MD5 Has Four Rounds

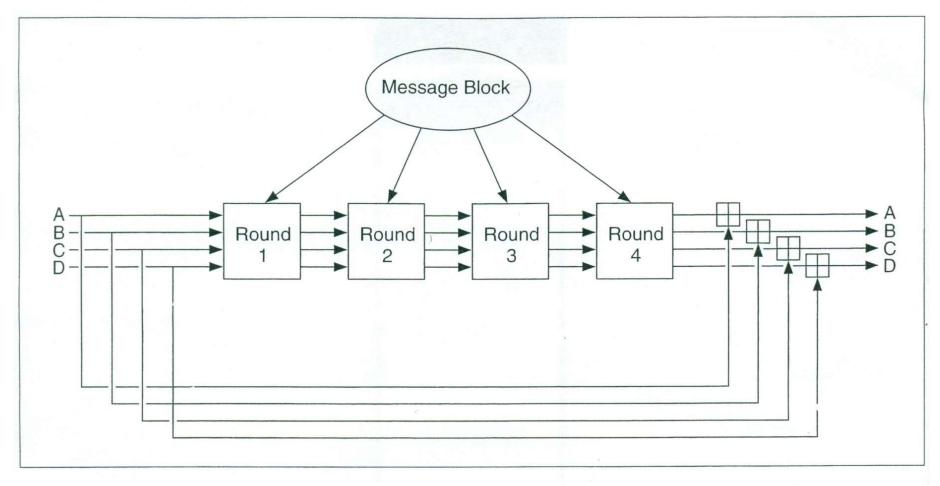


Figure 18.5 MD5 main loop. $_{\odot Georgia\ Institute\ of\ Technology,\ 2018-2025}$

A, B, C and D

- Four 32-bit variables are used in MD5
 - Referred to in the literature as *chaining variables*
- Initial values
 - A = 0x01234567
 - B = 0x89abcdef
 - C = 0xfedcba98
 - D = 0x76543210

MD5 Operation

- Initial values
 - a = A = 0x01234567
 - b = B = 0x89abcdef
 - c = C = 0xfedcba98
 - d = D = 0x76543210
- Each MD5 is a series of MD5 operations

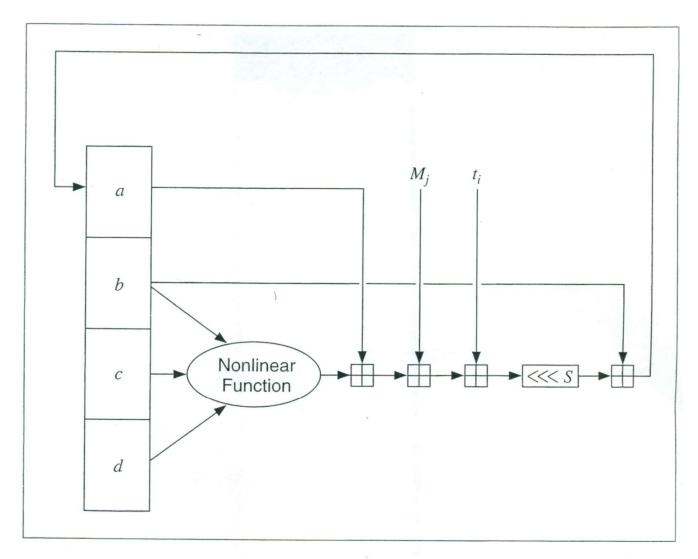


Figure 18:6811 Instruct Min 680 perention.

MD5 First Round

- FF(a,b,c,d,M_i,s,t_i) is defined as follows
 - $a = b + ((a + F(b,c,d) + M_i + t_i) <<< s)$
 - where the following definitions hold
 - F(X,Y,Z) = (X OR Y) AND ((notX) OR Z)
 - M_i represents the jth sub-block
 - 512 bit block divided into 16 sub-blocks each with 32 bits
 - For the 64 steps 16 per round t_i is the integer part of 2^{32*} abs(sin(i)) where i is in radians
 - Note that each t_i is a constant
 - << s represents a left circular shift of s bits

Round 1:

FF (a, b, c, d, M₀, 7, 0xd76aa478)

FF (d, a, b, c, M₁, 12, 0xe8c7b756)

FF $(c, d, a, b, M_2, 17, 0x242070db)$

FF $(b, c, d, a, M_3, 22, 0xc1bdceee)$

FF (a, b, c, d, M₄, 7, 0xf57c0faf)

FF (d, a, b, c, M₅, 12, 0x4787c62a)

FF (c, d, a, b, M₆, 17, 0xa8304613)

FF (b, c, d, a, M₇, 22, 0xfd469501)

FF (a, b, c, d, M₈, 7, 0x698098d8)

FF (d, a, b, c, M₉, 12, 0x8b44f7af)

FF (c, d, a, b, M₁₀, 17, 0xffff5bb1)

FF (b, c, d, a, M₁₁, 22, 0x895cd7be)

FF $(a, b, c, d, M_{12}, 7, 0x6b901122)$

FF $(d, a, b, c, M_{13}, 12, 0xfd987193)$

FF (c, d, a, b, M₁₄, 17, 0xa679438e)

FF (b, c, d, a, M₁₅, 22, 0x49b40821)

MD5 Second, Third and Fourth Rounds

- MD5 second, third and fourth rounds are very similar to the first
- Definitions of GG and G (second round), HH and H (third round), and II and I (fourth round) are provided by Schneier

MD5 Security Analysis

- Each step adds in the result of the previous step
 - Avalanche effect, i.e., the dependency of the output bits on the input bit spreads faster
- Left circular shifts also increase the avalanche effect
 - Diffusion

MD5 Hacks

- Believed to be collision resistant for many years...
 - "...in 2004 a team of Chinese cryptanalysts presented a new method finding collisions in MD5..." pg. 249 of Katz & Lindell
 - Flame malware attack discovered in 2012 used MD5 signatures to falsify a certificate claiming that code was from a legitimate company...

Collision Experiment on Hash Functions

- Note that as defined a hash function H maps a larger number of bits M into a smaller number of bits h
 - Therefore it is impossible to always generate a unique h
 - H may also be called or referred to as a compression function
- Collision-finding experiment
 - Adversary A finds a collision if A can find M and M' such that H(M') = H(M)
 - If it is infeasible for A to find a collision, we say that H is collision resistant

Weaker Notions of Security

Target-collision resistance

- Given a uniformly random M, it is infeasible for an adversary to find M' such that H(M') = H(M)
- Note1: this is also referred to in the literature as second preimage resistance
- Note2: collision resistance (see previous page) implies *target-collision* resistance, i.e., second preimage resistance

• Preimage resistance

- Given a uniformly random h, it is infeasible for an adversary to find M such that H(M) = h
- Note that second preimage resistance implies

NOTE: PLEASE SEE LECTURE RECORDING FOR ANY CONTENT WHICH MAY (OR MAY NOT!) HAVE BEEN ADDED TO THE EMPTY PAGES