

Cryptography Part II: Birthday Attack

*Cryptographic Hardware for
Embedded Systems
ECE 3170 A*

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Assoc. Prof. Vincent John Mooney III

Georgia Institute of Technology

How Many People Have Your Birthday?

- Assume that birthdays are randomly distributed throughout the year
 - E.g., 9 months after Aug. 29 is an equally likely birthday as any other day
 - Further assume Feb. 29 is excluded
- You walk into a room; how many people need to be in the room for there to be a 50% chance that one person has the same birthday as you?
- The chance that one person has the same birthday as you is $1/365$
- The second person may have the same birthday as you *or* the first person
⇒ the increase in probability including the second person is not $1/365$
- To get to approximately 50%, need to have 253 people in the room

What Are The Chances That Any Two People in a Room Have the Same Birthday?

- Two people: $1/365$
- Three people A, B and C: $A\&B = 1/365$, $A\&C = 1/365$, $B\&C = 1/365$
 \Rightarrow a total chance of $3/365$
- Four people A, B, C and D: $A\&B$, $A\&C$, $A\&D$, $B\&C$, $B\&D$, $C\&D$
 \Rightarrow a total chance of $6/365$
- Clearly, growth of chances is more than linear (the growth is polynomial)
- Final result: with 23 people in the room, the chance that two people share the same birthday is approximately 50%

What Does the Birthday Attack Illustrate?

- The difference between the chances of randomly finding one particular secret, e.g., a match to a specific person's birthday or the access code for a specific device
- Versus the chances of finding a collision, e.g., in a collection of devices two that have the same access code (key) or in a group of people any two who have the same birthday