Cryptography Part VII: CCA, HMAC and Unforgeability

hash tags

ECE 4156/6156 Hardware-Oriented Security and Trust

Spring 2024

Assoc. Prof. Vincent John Mooney III

Georgia Institute of Technology

Reading

- Introduction to Modern Cryptography, Chapter 3.7 (CCA-Security), Chapter 4, Chapter 5.1 and Chapter 5.3

Notation

plaintext = 128 × 64 → approx same size
2^7 2^6
tag = 128 bits $\sim 2^{13}$

- π_E = (Gen, Enc, Dec) is an encryption scheme
- π_M = (Gen, Mac, Vrfy) is a message authentication code or MAC
- Probabilistic Polynomial Time or PPT refers to algorithms which take at most polynomial time while having free use of a true random number generator

chosen ciphertext attacks

- $\text{PrivK}^{\text{cca}}_{A,\pi}(n)$ is an experiment involving a private key encryption scheme π with a key of size *n* and a PPT adversary A with access to ciphertext, an encryption oracle (without limits other than time) and a decryption oracle (but the challenge ciphertext may not be submitted)
- $H^s(x) \stackrel{\text{def}}{=} H(s, x)$ where the keyed hash function take inputs *s* and x in order to produce output *h*
  - A superscript is used for *s*, i.e., H^s, instead of a subscript, i.e., H_s in order to emphasize the fact that the typical attack surface includes scenarios where the adversary may have possession of the key

CCA-Security

No hw
No Text

- For Chosen Ciphertext Attack (CCA) security, the attacker has access to a decryption oracle
  - Experiment $\mathrm{PrivK}_{A,\pi}^{\mathrm{cca}}(n)$ is run with two messages m_0 and m_1 encrypted to c_0 and c_1 where the adversary A has to guess which message was encrypted given only the corresponding encrypted ciphertext
  - For obvious reasons, the adversary may not submit c_0 or c_1 to the decryption oracle!
- Some practical situations where partial access to a decryption oracle exists occur when error messages are provided
  - Based on which error message occurs, a CCA may commence where, for example, incorrect padding allows one to correctly guess the value of a byte
  - Padding oracle attack! (not covered this year in ECE 4156 / 6156)

Message Authentication Code (MAC) Design

do not verify the authenticity of sender, e.g., entity auth.

- In Lecture 3, Intro to SHA-2, hash functions were introduced
  - Collision resistance (strongest)
  - Target-collision resistance (a.k.a. second preimage resistance)
  - Preimage resistance
- SHA-2 is keyless (or you can say that the initial conditions are fixed)
- However, is this lecture we will introduce the concept of a MAC which is a keyed hash
- In Lecture 4, Authentication I, it was observed that typically what is meant by "Message Authentication" in a MAC is in fact message integrity, i.e., verification that a message has not been altered after being sent

integrity = message has not been altered

128-bit sym K

K_E K_M

MAC Definition

- A Message Authentication Code (MAC) π_M is composed of three PPT functions Gen, Mac and Vrfy

2^{256}

verify

- As with an encryption scheme π_E, Gen generates a key
  - We will denote the key for π_M as $\boxed{k_M}$
  - As with symmetric key encryption, we assume that key $\boxed{k_M}$ is provided to both parties (e.g., Alice and Bob) without being revealed to the adversary
- $\text{Mac}_{k_M}(m)$ takes as input a message m and uses k_M to output a small tag t
- $\text{Vrfy}_{k_M}(m,t)$ takes as inputs message m and tag t
  - Vrfy_{k_M} uses k_M to output a '1' if tag t corresponds to message m
  - Otherwise Vrfy_{k_M} outputs a '0'

composed to msg. s.t.

$C = Enc_K(m)$, sha256(k)

Verification that a Message is Unaltered

- The concept of a verifier Vrfy can also, in principle, be applied to keyless hashes, e.g., SHA2 or SHA3
- For a keyless hash such as SHA2 it is assumed that the tag *t* and message *m* are not easily replaced in transit (since the adversary clearly can calculate a new tag!)
  - One possibility is to send tag *t* encrypted
- In this case there is no key k_M used to compute tag *t* given message *m*
- In this case (which is not included in Katz and Lindell!) Vrfy(*m*,*t*) verifies if the appropriate keyless hash when provided message *m* as input gives as output tag *t*
- Canonical verification occurs with deterministic MACs and keyless hashes when the verifier simply recomputes tag *t* and checks for equality

$$\big(\overset{IV}{c_0}, \overset{m_1}{c_1}, \overset{m_2}{c_2}, \overset{m_3}{c_3}, \cancel{c_4}, t\big) \rightarrow 128 * 3 = 384 \text{ bits}$$

$$|IV| = 128 \qquad |t| = 128$$

$$(c_0, c_1, c_2, c_3, t)$$

Q: Can adv. forge (create) a tag for a new input (message) and pass Vrfy?

Toward the formal definition, consider the following experiment for a message authentication code $\Pi_M = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$, an adversary $\mathcal{A}$, and value n for the security parameter:

A

1^n

Q = set of oracle queries = Mac queries

|Q| ≤ polynomial in n

(m, t)

Know Vrfy algorithm

Server

1^n K

Vrfy/K

The message authentication experiment $\mathsf{Mac\text{-}forge}_{\mathcal{A},\Pi}(n)$**:**

1. *A key k is generated by running* $\mathsf{Gen}(1^n)$.
2. *The adversary $\mathcal{A}$ is given input 1^n and oracle access to* $\mathsf{Mac}_k(\cdot)$. *The adversary eventually outputs (m, t). Let $\boxed{\mathcal{Q}}$ denote the set of all queries that $\mathcal{A}$ asked its oracle.*
3. $\mathcal{A}$ succeeds *if and only if (1)* $\mathsf{Vrfy}_k(m, t) = 1$ *and (2) $m \notin \mathcal{Q}$. In that case the output of the experiment is defined to be 1.*

Our vrfy construct
+ proof does not
apply to Replay

new m

Existentially Unforgeable under an Adaptive Chosen-Message Attack

π_M k_M A $|t| = n$

- Given π_{M} and adversary A, $\boxed{\mathrm{Mac\text{-}forge}_{A,\pi_{\mathrm{M}}}(n)}$ checks to see if A can come up with a valid MAC tag t given message *m* and oracle access to $\mathrm{Mac}_{k_{\mathrm{M}}}$ except that *m* may not be submitted to the oracle
  - The requirement that $m \notin Q$, where Q is the set of all oracle queries, enforces that *m* may not be submitted to the oracle
- A tag is *existentially unforgeable*[1] for an arbitrary message *m* if an adversary has only a negligible change of generating a valid tag *t* given only message *m* (and, of course, no access to key k_{M}, i.e., a keyless hash does not fit this experiment)
  - The *adaptive chosen-message attack*[1] refers to the adversary's ability to arbitrarily choose message *m* during the attack itself, e.g., by adding spaces or commas to a legal statement contained in a message
  - The oracle access of the attacker models the case where the attacker can induce some messages (other than *m*) and obtain their corresponding tags

[1] Page 112 of Katz and Lindell.

1 definition for tag "experi-ment"

MAC   KM

DEFINITION 4.2 *A message authentication code* π_M = (Gen, Mac, Vrfy) *is* **existentially unforgeable under an adaptive chosen-message attack***, or just* $\boxed{\textbf{secure}}$*, if for all* PPT *adversaries* A *there is a negligible function* negl *such that, for all n,*

$$\Pr\left[\text{Mac-forge}_{A,\pi_M}(n) = 1\right] \leq \text{negl}(n).$$

Replay Attacks

- repday attacks are not prevented
- need to protect against repby attalks in the overall protocol

Replay Attacks

- Note that as presented the verifier has no access to any kind of history or record of previous messages
- Without any notion of state, the protocols presented will not be able to prevent replay attacks
- In practice, the two most popular approaches to prevent replay attacks are (i) use of a counter and (ii) use of a timestamp
- Use of a counter has the problem of synchronization
- Use of a timestamp has the problem of slack or clock skew
  - Attacks that are "fast enough" (i.e., within acceptable skew) may succeed
- Katz and Lindell pages 113-114

if Gen omitted, assume key from uniform dist.

TRNG

CONSTRUCTION 4.18

Let $\Pi_E = (\mathsf{Enc}, \mathsf{Dec})$ be a private-key encryption scheme and let $\Pi_M = (\mathsf{Mac}, \mathsf{Vrfy})$ be a message authentication code, where in each case key generation is done by simply choosing a uniform n-bit key. Define a private-key encryption scheme $(\mathsf{Gen}', \mathsf{Enc}', \mathsf{Dec}')$ as follows:

- Gen': on input 1^n, choose independent, uniform $k_E, k_M \in \{0,1\}^n$ and output the key (k_E, k_M).
- Enc': on input a key (k_E, k_M) and a plaintext message m, compute $c \leftarrow \mathsf{Enc}_{k_E}(m)$ and $t \leftarrow \mathsf{Mac}_{k_M}(c)$. Output the ciphertext $\langle c, t \rangle$.
- Dec': on input a key (k_E, k_M) and a ciphertext $\langle c, t \rangle$, first check whether $\mathsf{Vrfy}_{k_M}(c,t) \stackrel{?}{=} 1$. If yes, then output $\mathsf{Dec}_{k_E}(c)$; if no, then output $\perp$.

3 Enc. then Auth.

"Error"

A generic construction of an authenticated encryption scheme.

Π_E

Π_M

Throughout, let $\Pi_E = (\mathsf{Enc}, \mathsf{Dec})$ be a CPA-secure encryption scheme and let $\Pi_M = (\mathsf{Mac}, \mathsf{Vrfy})$ denote a message authentication code, where key generation in both schemes simply involves choosing a uniform n-bit key. There are three natural approaches to combining encryption and message authentication using independent keys[4] $\boxed{k_E}$ and $\boxed{k_M}$ for Π_E and Π_M, respectively:

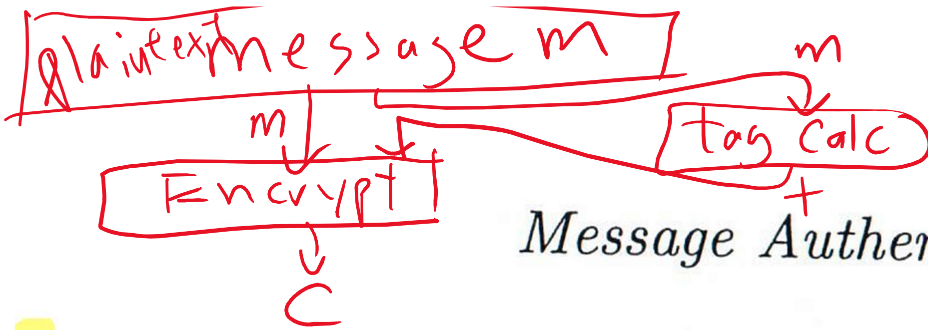
1. *Encrypt-and-authenticate:* In this method, encryption and message authentication are computed independently in parallel. That is, given a plaintext message m, the sender transmits the ciphertext $\langle c, t \rangle$ where:

$$c \leftarrow \mathsf{Enc}_{k_E}(m) \quad \text{and} \quad t \leftarrow \mathsf{Mac}_{k_M}(m).$$

$\langle c, t \rangle$

The receiver decrypts c to recover m; assuming no error occurred, it then verifies the tag t. If $\mathsf{Vrfy}_{k_M}(m, t) = 1$, the receiver outputs m; otherwise, it outputs an error.

[4] *Independent* cryptographic keys should always be used when different schemes are combined. We return to this point at the end of this section.



2. *Authenticate-then-encrypt:* Here a MAC tag t is first computed, and then the message and tag are encrypted together. That is, given a message m, the sender transmits the ciphertext c computed as:

$$t \leftarrow \mathsf{Mac}_{k_M}(m) \quad \text{and} \quad c \leftarrow \mathsf{Enc}_{k_E}(m \| t).$$

The receiver decrypts c to obtain $m\|t$; assuming no error occurs, it then verifies the tag t. As before, if $\mathsf{Vrfy}_{k_M}(m,t) = 1$ the receiver outputs m; otherwise, it outputs an error.

this works w/ SHA2

Constr. 4.18

3. *Encrypt-then-authenticate:* In this case, the message m is first encrypted and then a MAC tag is computed over the result. That is, the ciphertext is the pair $\langle c, t \rangle$ where:

$$c \leftarrow \mathsf{Enc}_{k_E}(m) \quad \text{and} \quad t \leftarrow \mathsf{Mac}_{k_M}(c).$$

$\langle c, t \rangle$

(See also Construction 4.18.) If $\mathsf{Vrfy}_{k_M}(c, t) = 1$, then the receiver decrypts c and outputs the result; otherwise, it outputs an error.

"clean" separation of encr. + auth.

Keyed MAC	Adv.	Dis adv.
1. Enc and ~~then~~ Auth <c,t>	run encr. + MAC in parallel (⇒ faster computation)	cryptanalysis on tag can potentially reveal info about m; must decrypt to test tag
2 Authenticate then Encrypt <c>	use SHA2 w/ one key	Decrypt yields m||t; cryptanalytic concern: m||t provides a relationship which may be exploited
3. Encrypt then Authenticate <c,T>	information leakage through the tag reveals nothing about the message	

Unkeyed ~~MAC~~ hash

tag generated on ciphertext can be recomputed by an adversary for change in any ciphertext block

=> option 3 will NOT work w/SHA2

Let's leave unkeyed MACs for later discussion ...

Later Discussion!

has no known successful attack methods. i.e., provide message integrity

- For a keyless hash intended to attest to the integrity of a message, which of the three approaches to combine encryption and message integrity are preferred and why?

SHA2 works?

- 1) Encrypt-and-authenticate
  - $c := Enc_k(m)$, $t := H^s(m)$, send <c,t>

Yes bec. if t changed, no way to change m s.t. $c = Enc_k(m)$

- 2) Authenticate-then-encrypt
  - $t := H^s(m)$, $c := Enc_k(m \parallel t)$, send <c>

Yes

- 3) Encrypt-then-authenticate
  - $c := Enc_k(m)$, $t := H^s(c)$, send <c,t>

No

DEFINITION 4.16 *A private-key encryption scheme* Π *is* unforgeable *if for all probabilistic polynomial-time adversaries* $\mathcal{A}$, *there is a negligible function* negl *such that:*

$$\Pr[\mathsf{Enc\text{-}Forge}_{\mathcal{A},\Pi}(n) = 1] \leq \mathsf{negl}(n).$$

Paralleling our discussion about verification queries following Definition 4.2, here one could also consider a stronger definition in which $\mathcal{A}$ is additionally given access to a decryption oracle. One can verify that the secure construction we present below also satisfies that stronger definition.

We now define a (secure) *authenticated encryption* scheme.

DEFINITION 4.17 *A private-key encryption scheme is an* authenticated encryption scheme *if it is CCA-secure and unforgeable.*

Forgeability Experiment

PPT

- Fix A, $\Pi_M(n)$
- Experiment $\text{Forge}_{A,\Pi_M}(n)$
  1. $K_M \leftarrow \text{Gen}(1^n)$
  2. Adversary A interacts w/ $\boxed{MAC_{K_M}}$ (let M be a set of messages submitted to the oracle)
  3. A outputs m, t
  4. A succeeds if $\text{Vrfy}(m,t) = 1$ and $m \notin M$

  A forges a valid tag

m chosen by A

t chosen by A

The unforgeable encryption experiment $\mathsf{Enc\text{-}Forge}_{\mathcal{A},\Pi}(n)$**:**

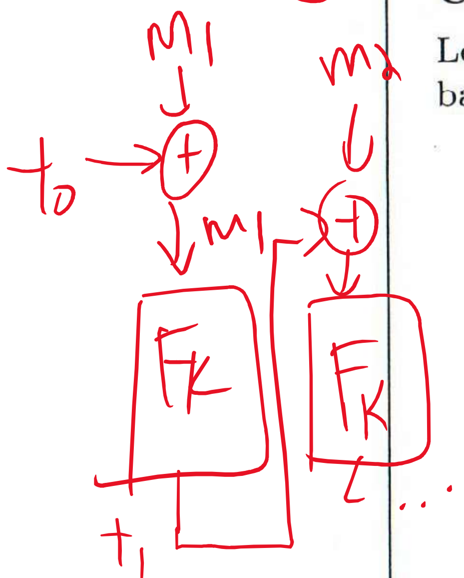
1. *Run* $\mathsf{Gen}(1^n)$ *to obtain a key* k_E.
2. *The adversary* $\mathcal{A}$ *is given input* 1^n *and access to an encryption oracle* $\mathsf{Enc}_{k_E}(\cdot)$. *The adversary outputs a ciphertext* c.
3. *Let* $m := \mathsf{Dec}_k(c)$, *and let* $\mathcal{Q}$ ← capital Q *denote the set of all queries that* $\mathcal{A}$ *asked its encryption oracle. The output of the experiment is* 1 *if and only if (1)* $m \neq \perp$ *and (2)* $m \notin \mathcal{Q}$.

4.4.1 The Basic Construction

CBC-MAC is a standardized message authentication code used widely in practice. A basic version of CBC-MAC, secure when authenticating messages of any *fixed* length, is given as Construction 4.11. (See also Figure 4.1.) We caution that this basic scheme is *not* secure in the general case when messages of different lengths may be authenticated; see further discussion below.

For 3. Encrypt then auth. the "message" is ciphertext?

CBC-MAC



CONSTRUCTION 4.11

Let F be a pseudorandom function, and fix a length function $\ell > 0$. The basic CBC-MAC construction is as follows:

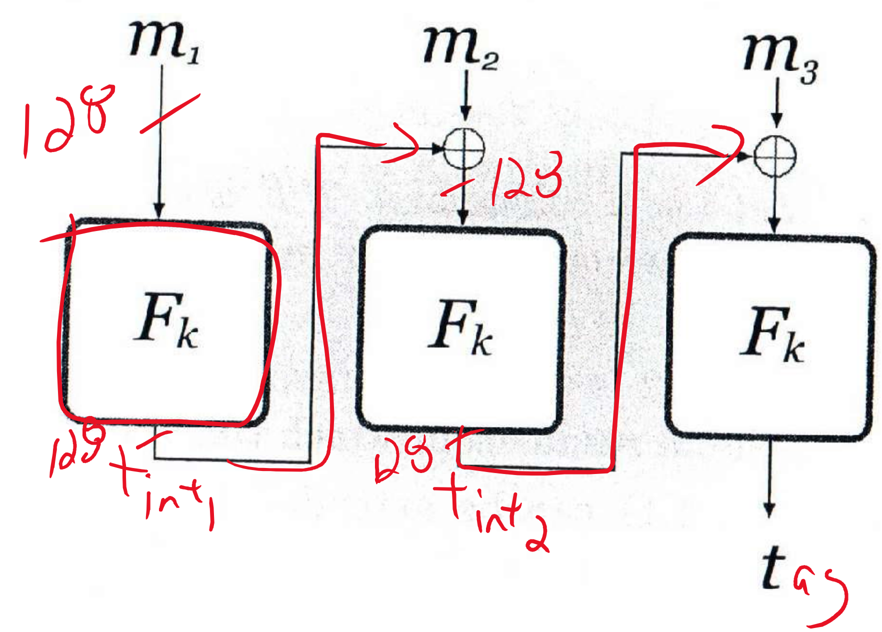
- Mac: on input a key $k \in \{0,1\}^n$ and a message m of length $\ell(n) \cdot n$, do the following (we set $\ell = \ell(n)$ in what follows):
  1. Parse m as $m = m_1, \ldots, m_\ell$ where each m_i is of length n.
  2. Set $t_0 := 0^n$. Then, for $i = 1$ to ℓ:
     Set $t_i := F_k(t_{i-1} \oplus m_i)$.

  Output t_ℓ as the tag.
- Vrfy: on input a key $k \in \{0,1\}^n$, a message m, and a tag t, do: If m is not of length $\ell(n) \cdot n$ then output 0. Otherwise, output 1 if and only if $t \stackrel{?}{=} \mathsf{Mac}_k(m)$.

Basic CBC-MAC (for fixed-length messages).

Vrfy runs CBC-MAC &

Secure CBC-MAC for arbitrary-length messages. We briefly describ two ways Construction 4.11 can be modified, in a provably secure fashior to handle arbitrary-length messages. (Here for simplicity we assume that a messages being authenticated have length a multiple of n, and that Vrfy reject



AES KE encrypt

FIGURE 4.1: Basic CBC-MAC (for fixed-length messages).

AES Km MAC

CONSTRUCTION 4.5

Let F be a pseudorandom function. Define a fixed-length MAC for messages of length n as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, output the tag $t := F_k(m)$. (If $|m| \neq |k|$ then output nothing.)
- Vrfy: on input a key $k \in \{0,1\}^n$, a message $m \in \{0,1\}^n$, and a tag $t \in \{0,1\}^n$, output 1 if and only if $t \stackrel{?}{=} F_k(m)$. (If $|m| \neq |k|$, then output 0.)

A fixed-length MAC from any pseudorandom function.

Not going to cover proof

THEOREM 4.6 *If F is a pseudorandom function, then Construction 4.5 is a secure fixed-length MAC for messages of length n.*

If we use (3) encrypt then authenticate

(i) symmetric key encr. scheme

(ii) message integrity w/MAC

$\Rightarrow$ combination of (i) and (ii)

(iii) is guaranteed to achieve authenticated encryption,

i.e., encr. has a negl. chance to be broken

and message integrity has negl. chance to be broken

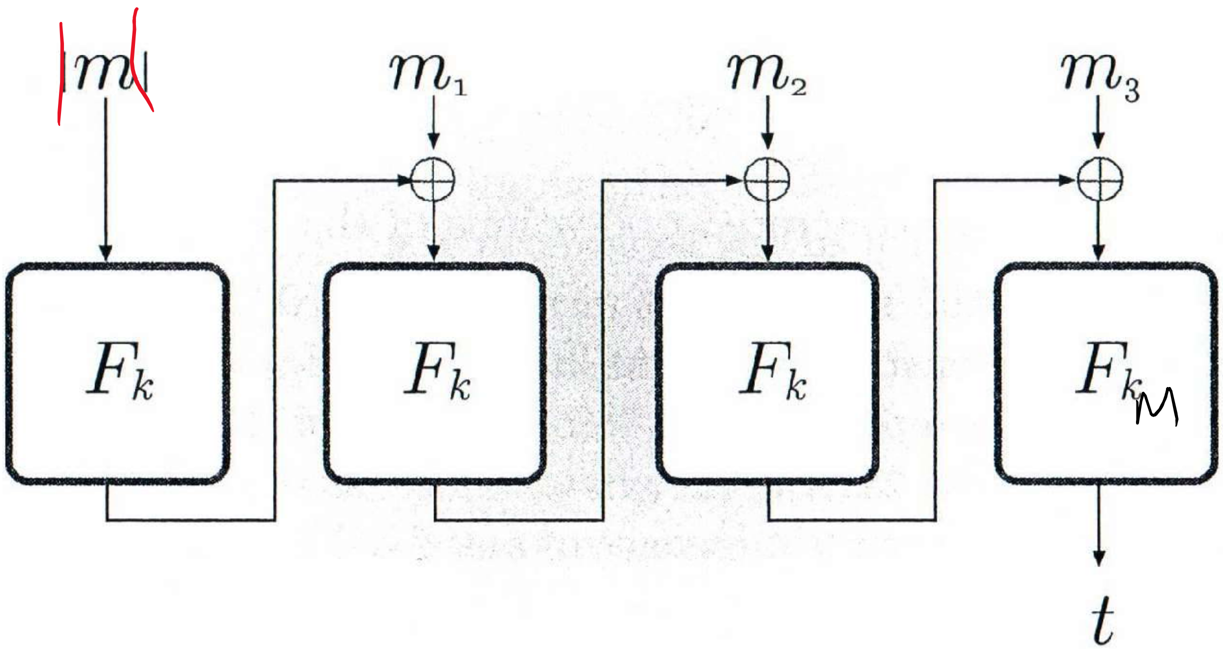


FIGURE 4.2: A variant of CBC-MAC secure for authenticating arbitrary-length messages.

Alice

K_E, K_M

Bob

K_E, K_M

AES

Π_E — CBC w/IV

CTR w/IV

AES

Π_M — CBC — Fixed length

variable length

Not cover

CONSTRUCTION 4.7

Let $\Pi' = (\mathsf{Mac}', \mathsf{Vrfy}')$ be a fixed-length MAC for messages of length n. Define a MAC as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^*$ of (nonzero) length $\ell < 2^{n/4}$, parse m into d blocks $m_1, \ldots, m_d$, each of length $n/4$. (The final block is padded with 0s if necessary.) Choose a uniform identifier $r \in \{0,1\}^{n/4}$.

  For $i = 1, \ldots, d$, compute $t_i \leftarrow \mathsf{Mac}'_k(r\|\ell\|i\|m_i)$, where i, ℓ are encoded as strings of length $n/4$.† Output the tag $t := \langle r, t_1, \ldots, t_d \rangle$.

- Vrfy: on input a key $k \in \{0,1\}^n$, a message $m \in \{0,1\}^*$ of length $\ell < 2^{n/4}$, and a tag $t = \langle r, t_1, \ldots, t_{d'} \rangle$, parse m into d blocks $m_1, \ldots, m_d$, each of length $n/4$. (The final block is padded with 0s if necessary.) Output 1 if and only if $d' = d$ and $\mathsf{Vrfy}'_k(r\|\ell\|i\|m_i, t_i) = 1$ for $1 \le i \le d$.

† Note that i and ℓ can be encoded using $n/4$ bits because $i, \ell < 2^{n/4}$.

A MAC for arbitrary-length messages from any fixed-length MAC.

Alice

Bob

K

SHA2

K

(2) Auth. then enc.

NOT BEEN BROKEN

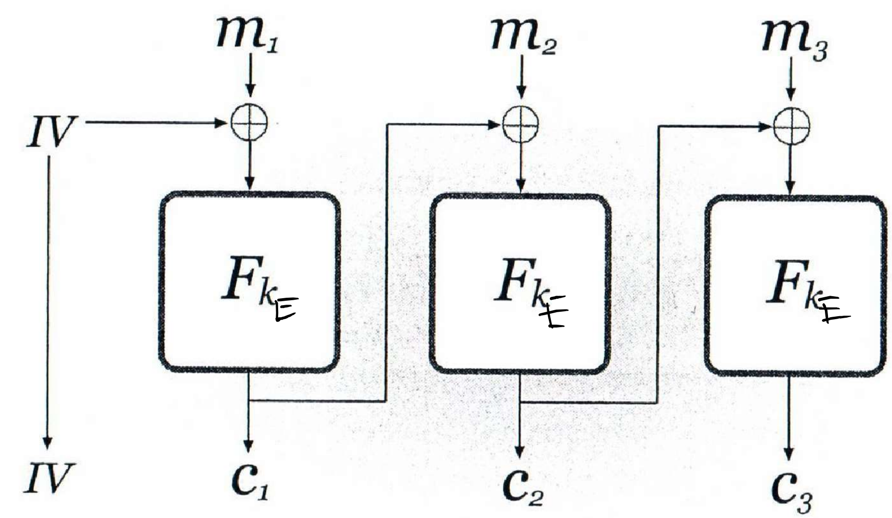


FIGURE 3.7: Cipher Block Chaining (CBC) mode.

Cipher Block Chaining (CBC) mode. To encrypt using this mode, a uniform initialization vector (IV) of length n is first chosen. Then, ciphertext blocks are generated by applying the block cipher to the XOR of the current plaintext block and the previous ciphertext block. That is, set $c_0 := IV$ and then, for $i = 1$ to ℓ, set $c_i := F_k(c_{i-1} \oplus m_i)$. The final ciphertext is $\langle c_0, c_1, \ldots, c_\ell \rangle$. (See Figure 3.7.) Decryption of a ciphertext $c_0, \ldots, c_\ell$ is done

$\langle c_0, c_1, c_2, c_3, t \rangle$

$128 \times 5 = 640$ bits