Cryptography Part IV: Encryption Modes

ECE 4156/6156 Hardware-Oriented Security and Trust

Spring 2024

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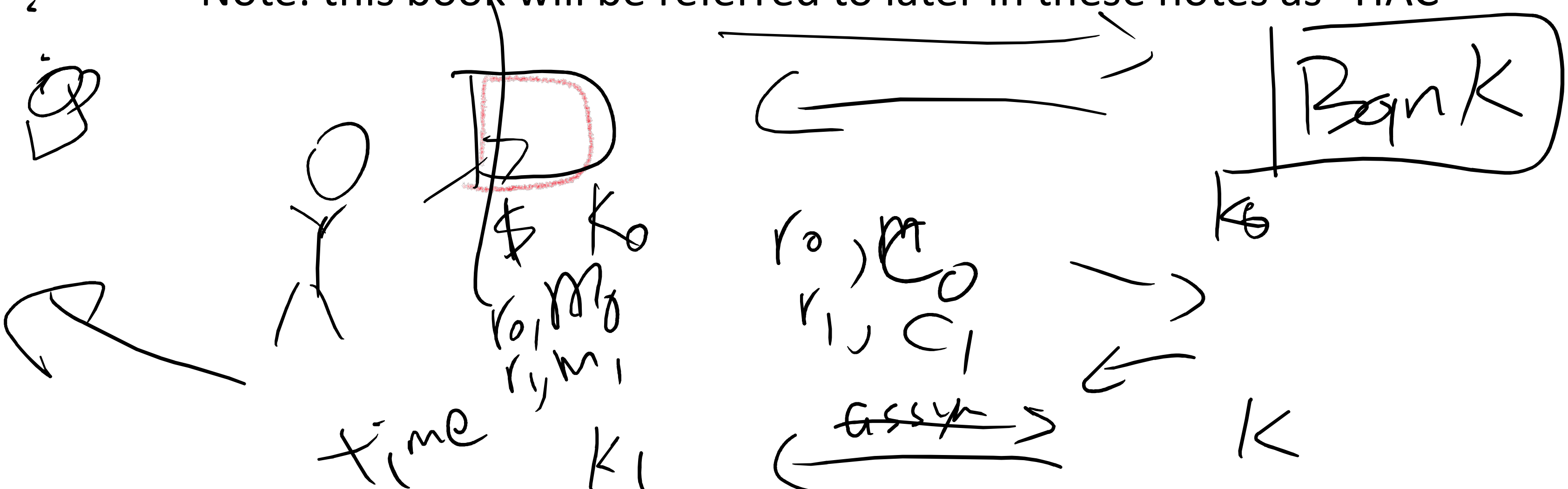
Reading Assignment

IV

- Please read Chapter 3 of the course textbook by Katz and Lindell
- Please read Chapter 2 of the course textbook by Menezes, Oorschot and Vanstone, i.e., the Handbook of Applied Cryptography
  - Note: this book will be referred to later in these notes as “HAC”



Notation from HAC (pages 49 and 50)

$B = \{0, \ldots, 2^{129}-1\}$

$A = \{2^{128}-1\}$

- $\mathbb{R}$ is the set of real numbers, e.g., $\pi \in \mathbb{R}$ while $\sqrt{-1} \notin \mathbb{R}$
- $\mathbb{Z}$ is the set of integers, i.e., $\mathbb{Z} = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$
- $f: A \to B$ is a function that maps each $a \in A$ to precisely one $b \in B$. Given that $f(a) = b$, then b is called the *image of* a, and a is called the *preimage* of b. The set A is called the *domain* of f.

  B is the range

- A function $f: A \to B$ is $1-1$ (*one-to-one*) or *injective* if each element in B is the image of at most one element in A. Hence $f(a_1) = f(a_2)$ implies $a_1 = a_2$.
- A function $f: A \to B$ is *onto* or *surjective* if each $b \in B$ is the image of at least one $a \in A$.
- A function $f: A \to B$ is a *bijection* if it is both one-to-one and onto. If f is a bijection between finite sets A and B, then $|A| = |B|$. If f is a bijection between a set A and itself, then f is called a *permutation* on A.

$f : A \to A$

other texts

Additional Notation (from Prof. Mooney)

- ℕ is the set of natural numbers, i.e., ℕ = {1,2,3,…}
- $f: A \to B$ is a function that maps each $a \in A$ to precisely one $b \in B$. Given that $f(a) = b$, then b is called the *image* of a, and a is called the *preimage* of b. The set A is called the *domain* of f. The set B is called the *range* of f.

Notation from Katz and Lindell

- $\{X\}$ is a set of elements of type X
- m is a message in plaintext
  - m is composed of smaller blocks m_i suitable for individual encryption steps
  - $m = \{m_i\}$
- c_i is ciphertext corresponding to message block m_i
- c is ciphertext corresponding to message m
- Enc_k is encryption with key k
  - $c \leftarrow Enc_k(m)$ (NOTE: there may be multiple valid ciphertexts!!!)
  - $c := Enc_k(m)$ (NOTE: deterministic, i.e., there is only one valid ciphertext)
- Dec_k is decryption with key k
  - $m := Dec_k(c)$ (NOTE: deterministic, i.e., there is only one valid message)
- $<a,b>$ is a concatenation of a followed by b
- $a||b$ is unambiguous concatenation of a followed by b; "unambiguous concatenation" means that a and b can be recovered from $a||b$

$a \,\|\, b$

$a_1 a_0 \,\|\, b_1 b_0$

for n bits

factorial

$$\left(\begin{array}{ll} a_1 a_0 b_1 b_0 & a_1 a_0 b_0 b_1 \\ a_1 b_1 a_0 b_0 & a_1 b_1 b_0 a_0 \\ a_1 b_0 a_0 b_1 & a_1 b_0 b_1 a_0 \end{array} \right)$$

$$(2^n)!$$

$$6 \times 4 = 24$$

Notation from Katz and Lindell (continued)

m_0 m_1 $b=1$

$c = Enc_k(m_1)$

if $b=0$

$c = Enc_k(m_0)$

- PrivK is an experiment involving a private key
- A is an adversary
- eav refers to eavesdropping and obtaining ciphertext only
- π = (Gen, Enc, Dec) is an encryption scheme
- $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}$ is an experiment involving a private key encryption scheme π with an adversary A only with access to ciphertext
- $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}(n)$ is an experiment involving a private key encryption scheme π with a key of size n and an adversary A only with access to ciphertext
- $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}(n,0)$ is an experiment involving a private key encryption scheme π with a key of size n, message selection bit b=0 and an adversary A only with ciphertext[1]

EAV

- A does not have access to additional information, e.g., A does not have valid plaintext-ciphertext pairs obtained through other means
- Probabilistic Polynomial Time or PPT refers to algorithms which take at most polynomial time while having free use of a true random number generator

[1] Page 55 of Katz and Lindell.

Recall Slide 11 from Crypto I Lecture

- M is a set of all possible messages, i.e., the message space
- C is a set of all possible ciphertexts, i.e., the ciphertext space
- *Gen* is a key generation procedure
  - The output of *Gen* is key k
  - *Gen* may or may not require an input

Now We Add the Following

- K is a set of all possible keys, i.e., the key space
- In the one-time pad, $|K| = |M| = |C| = \ell$

Where We Are So Far: Status

DEFINITION 2.5 *Encryption scheme* $\pi = (\text{Gen}, \text{Enc}, \text{Dec})$ *with message space* $\mathcal{M}$ *is* **perfectly indistinguishable** *if for every* A *it holds that*

$$\Pr\left[\text{PrivK}^{\text{eav}}_{A,\pi} = 1\right] = \frac{1}{2}.$$

Where We Are So Far: Status

DEFINITION 2.5 *Encryption scheme* π = (Gen, Enc, Dec) *with message space* M *is* **perfectly indistinguishable** *if for every* A *it holds that*

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DEFINITION 3.8 *A private-key encryption scheme* π = (Gen, Enc, Dec) *has* **indistinguishable encryptions in the presence of an eavesdropper**, *or is* **EAV-secure**, *if for all* PPT *adversaries* A *there is a negligible function* negl *such that, for all n,*

$$\Pr\left[\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n) = 1\right] \leq \frac{1}{2} + \mathrm{negl}(n),$$

$\frac{1}{2^{c}}$

where the probability is taken over the randomness used by A *and the randomness used in the experiment (for choosing the key and bit b, as well as any randomness used by* Enc*).*

Where We Are So Far: Status

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Where We Are So Far: Status (continued)

DEFINITION 3.9 *A private-key encryption scheme* π = (Gen, Enc, Dec) *has* **indistinguishable encryptions in the presence of an eavesdropper** *if for all* PPT *adversaries* A *there is a negligible function* negl *such that*

$$\left|\Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n,0)) = 1\right] - \Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n,1)) = 1\right]\right| \leq \mathrm{negl}(n).$$

$$\left|\Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{EAV}}_{A,\pi}(n,0)) = 0\right] - \Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n,1)) = 0\right]\right| \leq \mathrm{negl}(n)$$

Where We Are So Far: Status (continued)

$m_1 \quad m_0$

$6 = ?$

DEFINITION 3.9 *A private-key encryption scheme* π = (Gen, Enc, Dec) *has* **indistinguishable encryptions in the presence of an eavesdropper** *if for all* PPT *adversaries* A *there is a negligible function* negl *such that*

C

$$\left|\Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n,0)) = 1\right] - \Pr\left[\mathrm{out}_A(\mathrm{PrivK}^{\mathrm{eav}}_{A,\pi}(n,1)) = 1\right]\right| \le \mathrm{negl}(n).$$

$\ell = 128$

THEOREM 3.10 *Let* π = (Enc, Dec) *be a fixed-length private-key encryption scheme for messages of length* ℓ *that has indistinguishable encryptions in the presence of an eavesdropper. Then for all* PPT *adversaries* A *and any* $i \in \{1,\ldots,\ell\}$, *there is a negligible function* negl *such that*

$$\Pr\left[A(1^n, \mathrm{Enc}_k(m)) = m^i\right] \le \frac{1}{2} + \mathrm{negl}(n),$$

m^{19}

where the probability is taken over uniform $m \in \{0,1\}^\ell$ *and* $k \in \{0,1\}^n$, *the randomness of* A, *and the randomness of* Enc.

unknown $\quad m_3 \quad$ get m_3^{19}

Where We Are So Far: Status (continued)

THEOREM 3.10 *Let* π = (Enc, Dec) *be a fixed-length private-key encryption scheme for messages of length* ℓ *that has indistinguishable encryptions in the presence of an eavesdropper. Then for all* PPT *adversaries* A *and any* $i \in \{1,\ldots,\ell\}$, *there is a negligible function* negl *such that*

$$\Pr\left[A(1^n, \mathrm{Enc}_k(m)) = m^i\right] \leq \frac{1}{2} + \mathrm{negl}(n),$$

where the probability is taken over uniform $m \in \{0,1\}^\ell$ *and* $k \in \{0,1\}^n$, *the randomness of* A, *and the randomness of* Enc.

DEFINITION 3.14 *Let ℓ be a polynomial and let G be a deterministic polynomial-time algorithm such that for any n and any input $s \in \{0,1\}^n$, the result $G(s)$ is a string of length $\ell(n)$. We say that G is a* pseudorandom generator *if the following conditions hold:*

1. **(Expansion:)** *For every n it holds that $\ell(n) > n$.*

2. **(Pseudorandomness:)** *For any* PPT *algorithm D, there is a negligible function* negl *such that*

$$\left| \Pr[D(G(s)) = 1] - \Pr[D(r) = 1] \right| \leq \mathsf{negl}(n),$$

   where the first probability is taken over uniform choice of $s \in \{0,1\}^n$ and the randomness of D, and the second probability is taken over uniform choice of $r \in \{0,1\}^{\ell(n)}$ and the randomness of D.

We call ℓ the expansion factor *of G.*

Framework

pseudorandom
bit stream
generator
stream cipher

$|y_i| = 1$

GetBits
not
Specified

ALGORITHM 3.16
Constructing G_ℓ from (Init, GetBits)

Input: Seed s and optional initialization vector IV
Output: $y_1, \ldots, y_\ell$

$\mathsf{st}_0 := \mathsf{Init}(s, IV)$
for $i = 1$ to ℓ:
  $(y_i, \mathsf{st}_i) := \mathsf{GetBits}(\mathsf{st}_{i-1})$
return $y_1, \ldots, y_\ell$

st_i

y_1
$\vdots$
y_ℓ

why is Π not CPA-secure?

CONSTRUCTION 3.17

Let G be a pseudorandom generator with expansion factor ℓ. Define a private-key encryption scheme for messages of length ℓ as follows:

- Gen: on input 1^n, choose uniform $k \in \{0,1\}^n$ and output it as the key.
- Enc: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^{\ell(n)}$, output the ciphertext
$$c := G(k) \oplus m.$$
- Dec: on input a key $k \in \{0,1\}^n$ and a ciphertext $c \in \{0,1\}^{\ell(n)}$, output the message
$$m := G(k) \oplus c.$$

A private-key encryption scheme based on any pseudorandom generator.

query m_0

THEOREM 3.18 *If G is a pseudorandom generator, then Construction 3.17 is a fixed-length private-key encryption scheme that has indistinguishable encryptions in the presence of an eavesdropper.*

PROOF Let Π denote Construction 3.17. We show that Π satisfies Definition 3.8. Namely, we show that for any probabilistic polynomial-time adversary $\mathcal{A}$ there is a negligible function negl such that

$$\Pr\left[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(n). \tag{3.2}$$

Result(s)

- Given a pseudorandom number generator (PRNG) *G*
  - An exact example has yet to be provided
  - Definition 3.14, however, provides a framework to evaluate pseudorandom number generators
  - A PRNG efficiently expands a uniform (random) seed into a much larger pseudorandom output
    - Keeping the output length under a specified length provides number sequences which have no currently known way to be efficiently distinguished from a truly random number sequence
    - After the length is reached, use a new seed; note also the seed should be large, e.g., 128 bits, so than an adversary cannot guess the seed with any non-negligible probability of success
    - The seeds should be generated by a truly random physical process
  - No formal proof that PRNG's exist has been provided; but many practical constructions exist that have passed the "test of time"
- Construction 3.17 defines an encryption scheme π using *G*
- Theorem 3.18 proves that Construction 3.17 is EAV-secure

box" that encrypts messages of $\mathcal{A}$'s choice using a key k that is unknown to $\mathcal{A}$. That is, we imagine $\mathcal{A}$ has access to an "oracle" $\mathsf{Enc}_k(\cdot)$; when $\mathcal{A}$ *queries* this oracle by providing it with a message m as input, the oracle returns a ciphertext $c \leftarrow \mathsf{Enc}_k(m)$ as the reply. (When Enc is randomized, the oracle uses fresh randomness each time it answers a query.) The adversary is allowed to interact with the encryption oracle adaptively, as many times as it likes.

Consider the following experiment defined for any encryption scheme $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$, adversary $\mathcal{A}$, and value n for the security parameter:

$\mathcal{A}$

$m_0 \; m_1$

c

polynomial (in n)

$c_i = \mathsf{Enc}_k(m_i)$

e.g., $i \le n^c$

for c a constant

$\tilde{b}$

The CPA indistinguishability experiment $\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n)$:

1. *A key k is generated by running* $\mathsf{Gen}(1^n)$.
2. *The adversary $\mathcal{A}$ is given input 1^n and oracle access to* $\mathsf{Enc}_k(\cdot)$, *and outputs a pair of messages m_0, m_1 of the same length.*
3. *A uniform bit $b \in \{0,1\}$ is chosen, and then a ciphertext $c \leftarrow \mathsf{Enc}_k(m_b)$ is computed and given to $\mathcal{A}$.*
4. *The adversary $\mathcal{A}$ continues to have oracle access to* $\mathsf{Enc}_k(\cdot)$, *and outputs a bit b'.*
5. *The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise. In the former case, we say that $\mathcal{A}$* succeeds.

Π

K

$m_0 \; m_1$

b

$c_0 \; c_1$

$c = c_b$

DEFINITION 3.22 *A private-key encryption scheme* $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ *has* indistinguishable encryptions under a chosen-plaintext attack, *or is* CPA-secure, *if for all probabilistic polynomial-time adversaries* $\mathcal{A}$ *there is a negligible function* negl *such that*

$$\Pr\left[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(n),$$

where the probability is taken over the randomness used by $\mathcal{A}$*, as well as the randomness used in the experiment.*

This Concludes Where We Are So Far!!!

Construction 3.17 is not CPA-secure

- Why?

Construction 3.17 is not CPA-secure

- Why?
- In the CPA indistinguishability experiment $\mathrm{PrivK}_{A,\pi}^{\mathrm{cpa}}(n)$ step 2 provides oracle access to $\mathrm{Enc}_k(\cdot)$
  - (see page 74 of Katz and Lindell for the full list of steps)
  - Note that even though key k is secret, the adversary nonetheless has access to $\mathrm{Enc}_k(\cdot)$
- In step 4 the adversary continues to have oracle access prior to issuing a decision
- Clearly the adversary can simply compute $\mathrm{Enc}_k(m_0)$ and $\mathrm{Enc}_k(m_1)$!

Keyed Functions[2]

K

key $\quad m \quad c \quad K$

- A keyed function $F\colon \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$ has two inputs where the first is the key *k*
- Typically the inputs and output all have the same size *n* , i.e., $* = n$ in above
  - Given key *k*, the keyed function is F_k
  - Then we have $F_k\colon \{0,1\}^n \rightarrow \{0,1\}^n$ where $\boxed{F_k(x) = F(k,x)}$

$K \leftarrow \{0,1\}^n$

$F\colon K \times M \rightarrow C$

$F_K(m)$

$AES(K, m_0)$

$= AES_K(m_0)$

[2] Page 77 of Katz and Lindell.

four balls: red, green, blue, yellow

n = 2 f(00) = pick a ball, y ⇒ f(00) = y

replace y f(01) = pick g f(01) = g

replace g f(10) = pick r f(10) = r

f(11) = y

replace r f(11) = pick a ball y

Pseudorandom Functions

- Keyed function F_k is a **pseudorandom function** if for all PPT distinguishers D the chance that D can distinguish F_k is from a uniform function f is negligible.[3]
  - Note that a uniform function is not necessarily bijective
    - ~~If $F_k: \{0,1\}^n \to \{0,1\}^n$, the comparable uniform function~~ $f: \{0,1\}^n \to \{0,1\}^n$ may possibly have $f(x) = f(y)$ for $x \neq y$ with probability $\frac{1}{2^n}$

$f(00) = y = f(11)$

[3] See Def. 3.25 on page 79 of Katz and Lindell.

four balls

3 balls $f(00) = \text{pick}, y$

2 balls $f(01) = \text{pick}, g$

$f(10) = \text{pick}$

$f(00) = y$
$f(01) = g$
$f(10) = r$
$f(11) = b$

Pseudorandom Permutation

- Keyed function $\boxed{F_k}$ is a **pseudorandom permutation** if for all PPT distinguishers D the chance that D can distinguish F_k is from a uniform permutation *f* is negligible.[4]
  - Function $f{:}\{0,1\}^n \rightarrow \{0,1\}^n$ is a uniform permutation if it is bijective.

- In practice, for sufficiently large n, the distinction between a uniform function and a uniform permutation is indistinguishable.[4]

[4] Page 80 of Katz and Lindell.

SHA2: $\{0,1\}^{an} \to \{0,1\}^n$ $f(x) = 6$

$a > 1$ $f(y) = 6$ $f^{-1}(6) = ?$ x y

- A uniform function $f\colon \{0,1\}^{n} \to \{0,1\}^n$ is deterministic, i.e., for each input the output is defined, known and does not change
- The inverse of a uniform function $f\colon \{0,1\}^n \to \{0,1\}^n$, i.e., $f^{-1}\colon \{0,1\}^n \to \{0,1\}^n$ is typically not going to be deterministic because there may be an input with multiple valid outputs
- The inverse of a uniform function $f\colon A \to B$, i.e., $f^{-1}\colon B \to A$ is typically not going to be deterministic because there may be an input with multiple valid outputs

$n = 128$

r_1 r_2

K is secret

$r_1 \neq r_2$

$F_K(\cdot)$

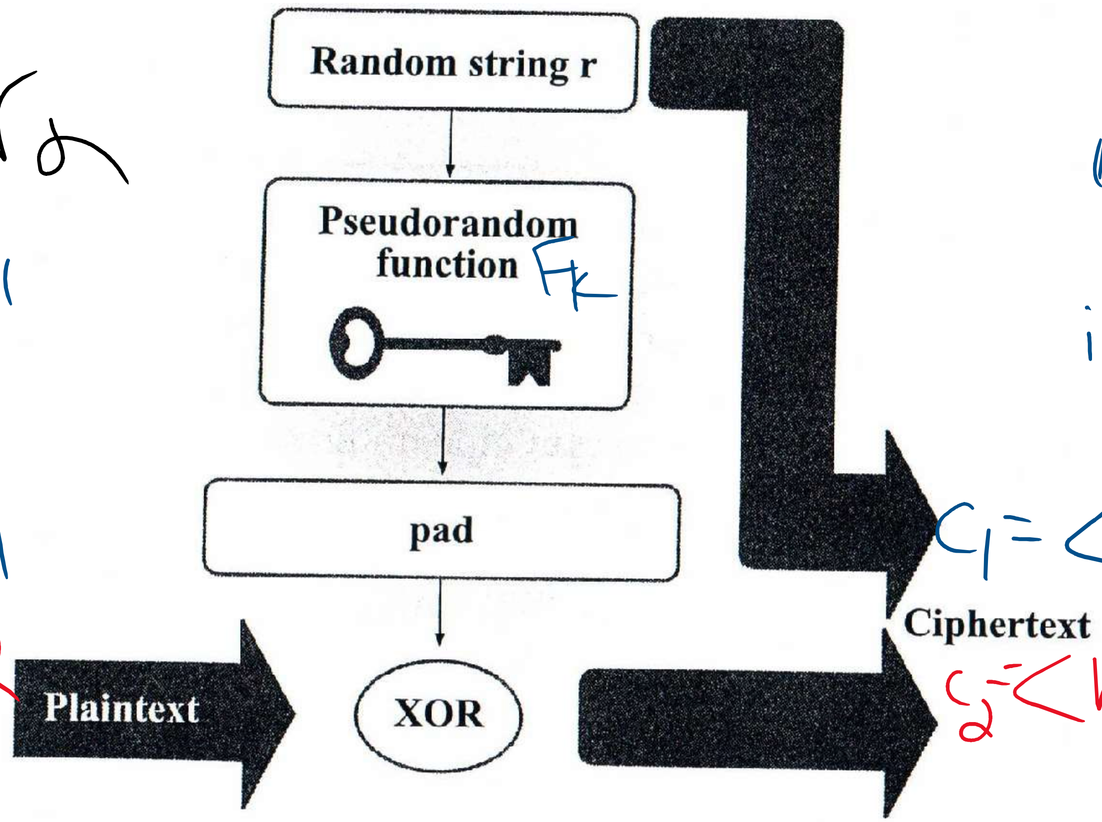
r_1 could be IV_1

in the clear

$\downarrow$

MIDWAY_1

MIDWAY_2



$c_1 = \langle r_1, F_K(r_1) \oplus \text{MIDWAY}_1 \rangle$

$c_2 = \langle r_2, F_K(r_2) \oplus \text{MIDWAY}_2 \rangle$

FIGURE 3.3: Encryption with a pseudorandom function.

$r_1 = r_2$ w/ probability $\frac{1}{2^{128}}$ $\quad \frac{128^3}{2^{128}}$

∃ Adv. GetBits

Stream cipher

Const. 3.29 F GetBits

s

n Init

IV

adv. knows IV part of st_0

IV'

n Init

st_0

IV

st_1

IV' = IV + 1

CONSTRUCTION 3.29

Let F be a pseudorandom function. Define a stream cipher (Init, GetBits), where each call to GetBits outputs n bits, as follows:

- Init: on input $s \in \{0,1\}^n$ and $IV \in \{0,1\}^n$, set $\mathsf{st}_0 := (s, IV)$.
- GetBits: on input $\mathsf{st}_i = (s, IV)$, compute $IV' := IV + 1$ and set $y := F_s(IV')$ and $\mathsf{st}_{i+1} := (s, IV')$. Output (y, st_{i+1}).

A stream cipher from any pseudorandom function/block cipher.

is $\frac{poly(n)}{2^n}$ negl.? Yes!

y

CONSTRUCTION 3.30

Let F be a pseudorandom function. Define a private-key encryption scheme for messages of length n as follows:

- Gen: on input 1^n, choose uniform $k \in \{0,1\}^n$ and output it.
- Enc: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, choose uniform $r \in \{0,1\}^n$ and output the ciphertext
$$c := \langle r,\ F_k(r) \oplus m \rangle.$$
- Dec: on input a key $k \in \{0,1\}^n$ and a ciphertext $c = \langle r, s \rangle$, output the plaintext message
$$m := F_k(r) \oplus s.$$

A CPA-secure encryption scheme from any pseudorandom function.

$n = 128$

$|c| = 256$

CONSTRUCTION 3.30

Let F be a pseudorandom function. Define a private-key encryption scheme for messages of length n as follows:

- Gen: on input 1^n, choose uniform $k \in \{0,1\}^n$ and output it.
- Enc: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, choose uniform $r \in \{0,1\}^n$ and output the ciphertext

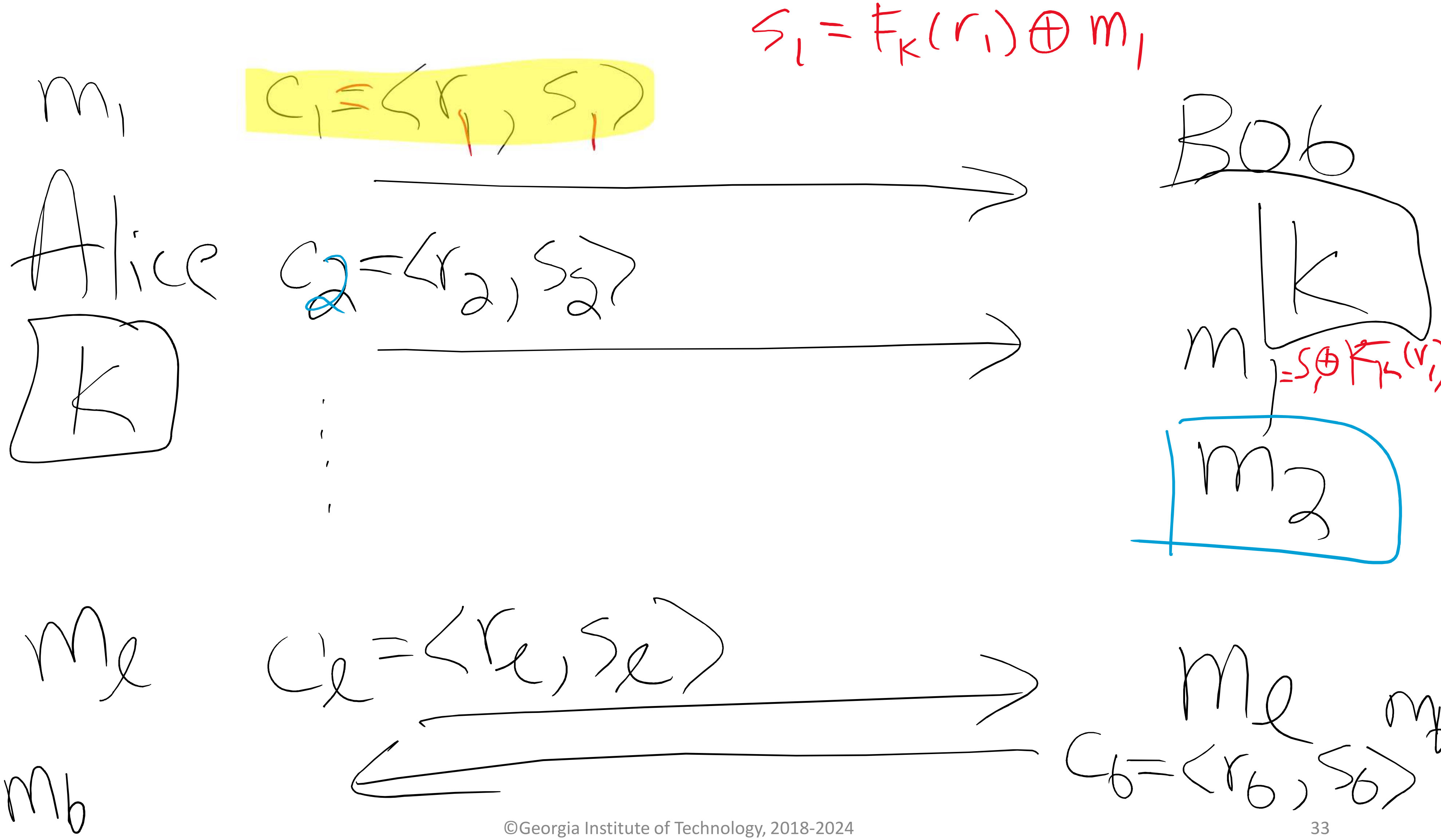
$$c := \langle r, F_k(r) \oplus m \rangle. = \langle r, s \rangle$$

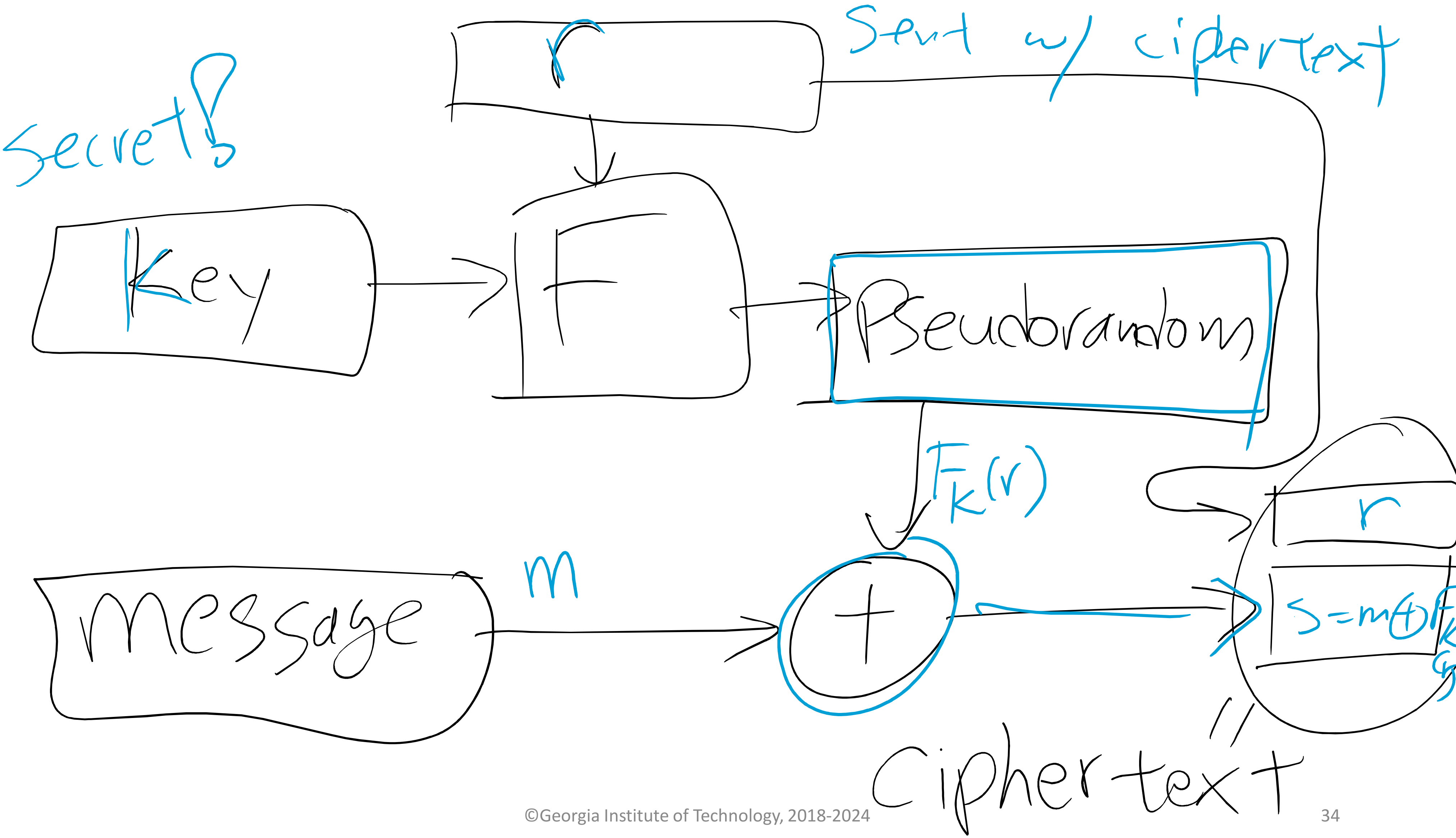
$s = F_k(r) \oplus m$

- Dec: on input a key $k \in \{0,1\}^n$ and a ciphertext $c = \langle r, s \rangle$, output the plaintext message

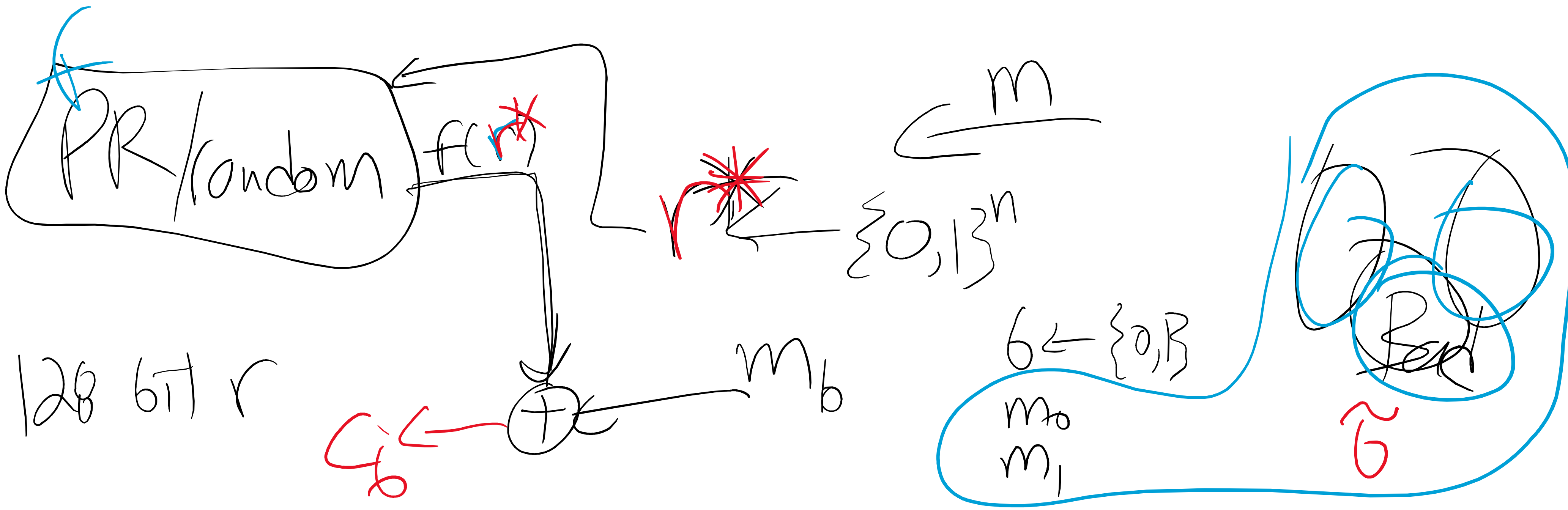
$$m := F_k(r) \oplus s.$$

A CPA-secure encryption scheme from any pseudorandom function.


$s_1 = F_k(r_1) \oplus m_1$
m_1
$c_1 = \langle r_1, s_1 \rangle$
Alice
Bob
k
$c_2 = \langle r_2, s_2 \rangle$
k
$m_1 = s_1 \oplus F_k(r_1)$
m_2
m_ℓ
$c_\ell = \langle r_\ell, s_\ell \rangle$
m_ℓ
m_b
$c_b = \langle r_b, s_b \rangle$
m_b


Secret!
Key
r
F
Sent w/ ciphertext
Pseudorandom
$F_K(r)$
message
m
+
r
$s=m\oplus F_K(r)$
ciphertext

ρ F np



$c_b \langle r^*, f(r^*) \oplus \cancel{m_b} \rangle$ negl $= \frac{\text{polynomial}}{2^{128}}$

r^* = case that r^* chosen = ciphertext

$\text{if } r^*_1 \neq r^*_0$, CPA-secure

Conclusion:
poly. sized set $\{r_a\}$ has $\frac{poly(n)}{2^n}$

Oracle

m_0

B

random

$(f_k(r_a)) \oplus {m_0 \atop m_1} = S_a$?

$C_a = \langle r_a, S_a \rangle$

$\begin{pmatrix} r_a \neq r^{*0} \\ r_a \neq r^{*1} \end{pmatrix}$

m_0, m_1

Ciphertext
$\langle r_a, S_a \rangle$

$f_{k\,guess}(r_a)$

candidate f_{cand}

EAV-secure

CPA-secure

EAV-secure

~~CPA-secure~~

~~EAV-secure~~

~~CPA-secure~~

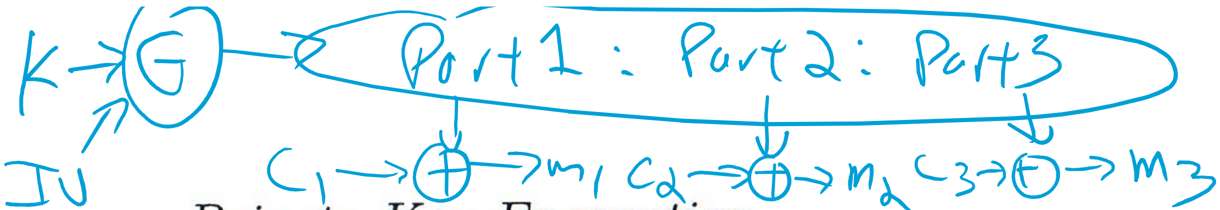
Given F is Pseudorandom, Construction 3.30 is CPA-secure

I am happy in office hours

- I hereby state the following:
- "The book goes through the proof in more detail, I just want you to get the intuition behind why Construction 3.30 is CPA-secure…I am not going to assign the proof on a homework or a test, guaranteed, …, however, **understanding** the intuition behind the proof is required and could be asked on a homework or a test!"

hw3: give examples, give intuition

e.g., the chance the $ra = r \star 0$ or $ra = p \star 1$ is essentially zero synch. decrypt



using Cunter. 3.29
stream cipher

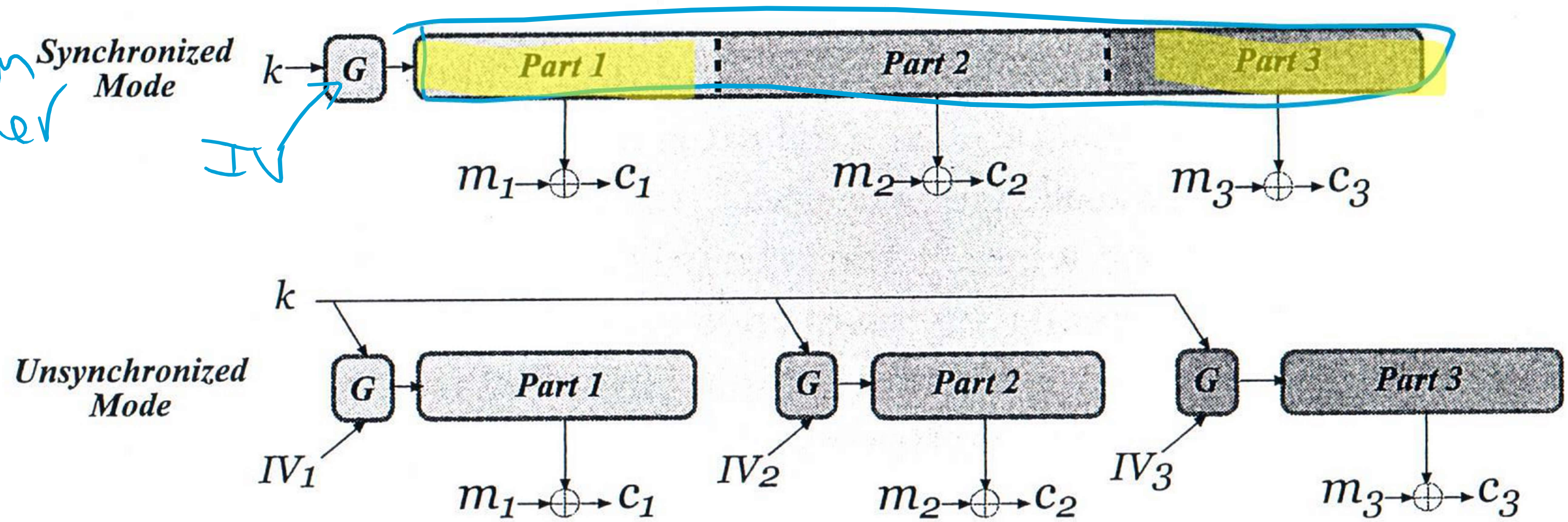
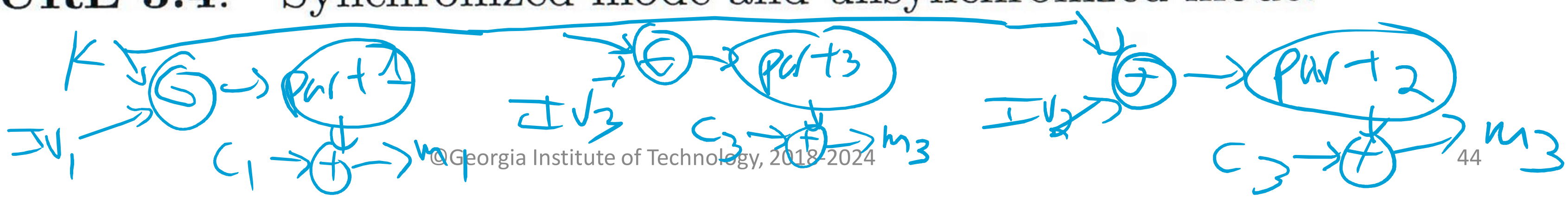
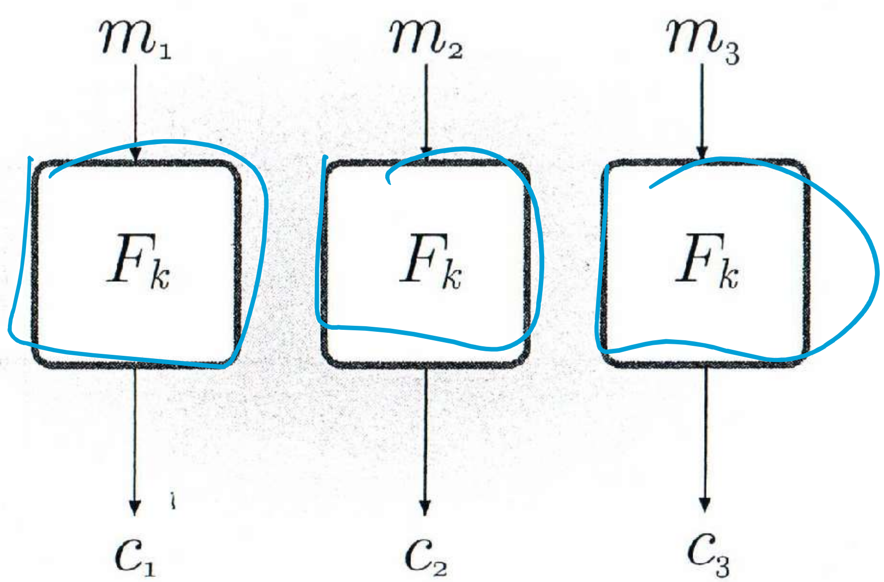


FIGURE 3.4: Synchronized mode and unsynchronized mode.

unsynch. decrypt



NIST 1977



not CPA-secure

FIGURE 3.5: Electronic Code Book (ECB) mode.

Figure 3.5. Decryption is done in the obvious way, using the fact that F_k^{-1} is efficiently computable.

$c_1 = Enc_k(m_1)$

$c_2 = Enc_k(m_2)$

$\vdots$

Problem: Deterministic?

if $c_i = c_j \Rightarrow m_i = m_j$

not provide indist. enc.

if (cond)
outp='1';
else outp='0'

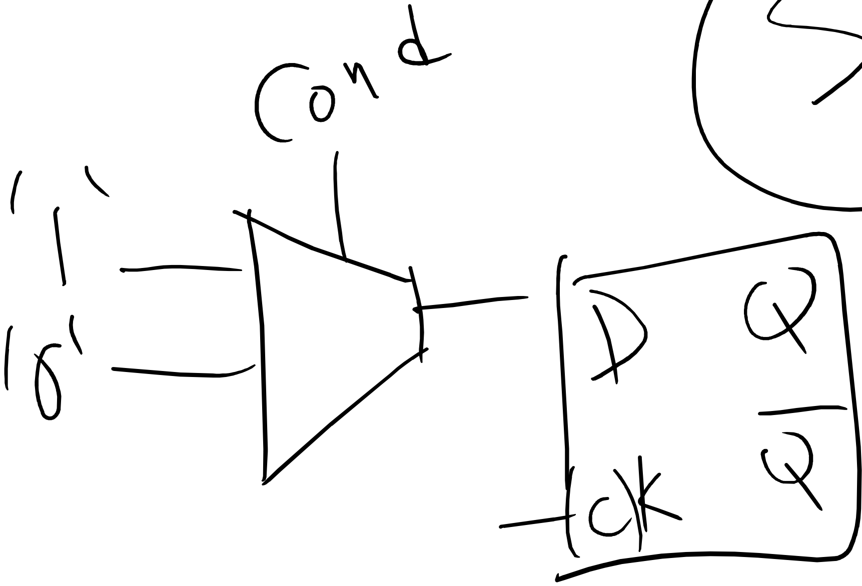
start =

suggest: create testbench for state machine

sha256 is picky

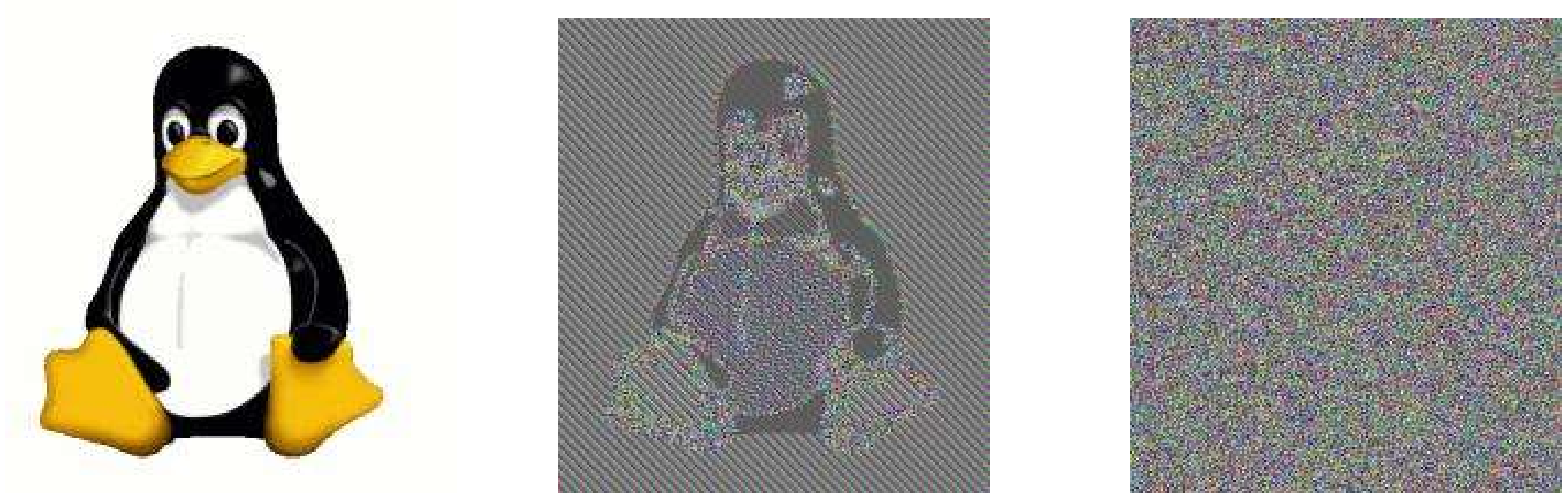
cx_i =

while s0, out
while s1,
while s2



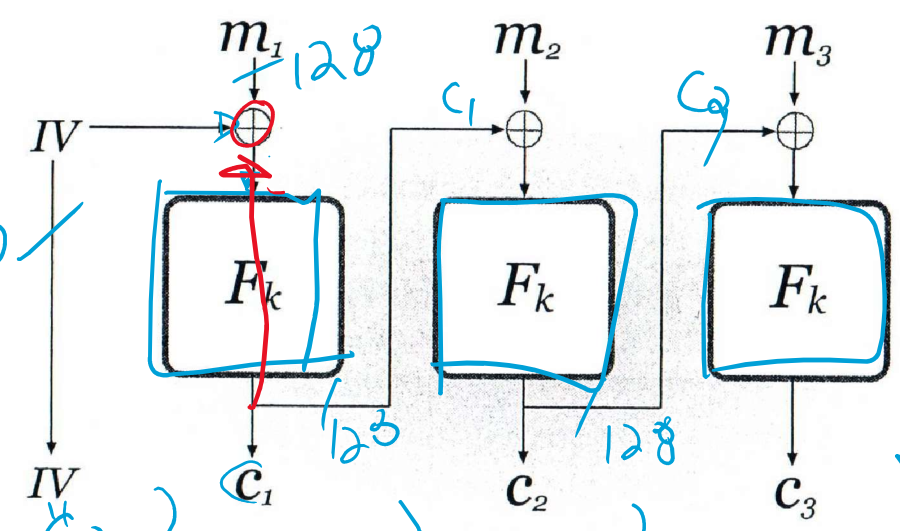
For these reasons, ECB mode should never be used. (We include it only because of its historical significance.)

FIGURE 3.6: An illustration of the dangers of using ECB mode. The middle figure is an encryption of the image on the left using ECB mode; the figure on the right is an encryption of the same image using a secure mode. (Taken from `http://en.wikipedia.org` and derived from images created by Larry Ewing (`lewing@isc.tamu.edu`) using The GIMP.)



[2] From https://en.wikipedia.org/wiki/Block_cipher_mode_of_operation and available under an open source license from Creative Commons.

3-block ciphertext



$C = \langle c_0, \; , \; , \; , \rangle$

$|m|$

$= |m_1, m_2, m_3|$

$= 384$

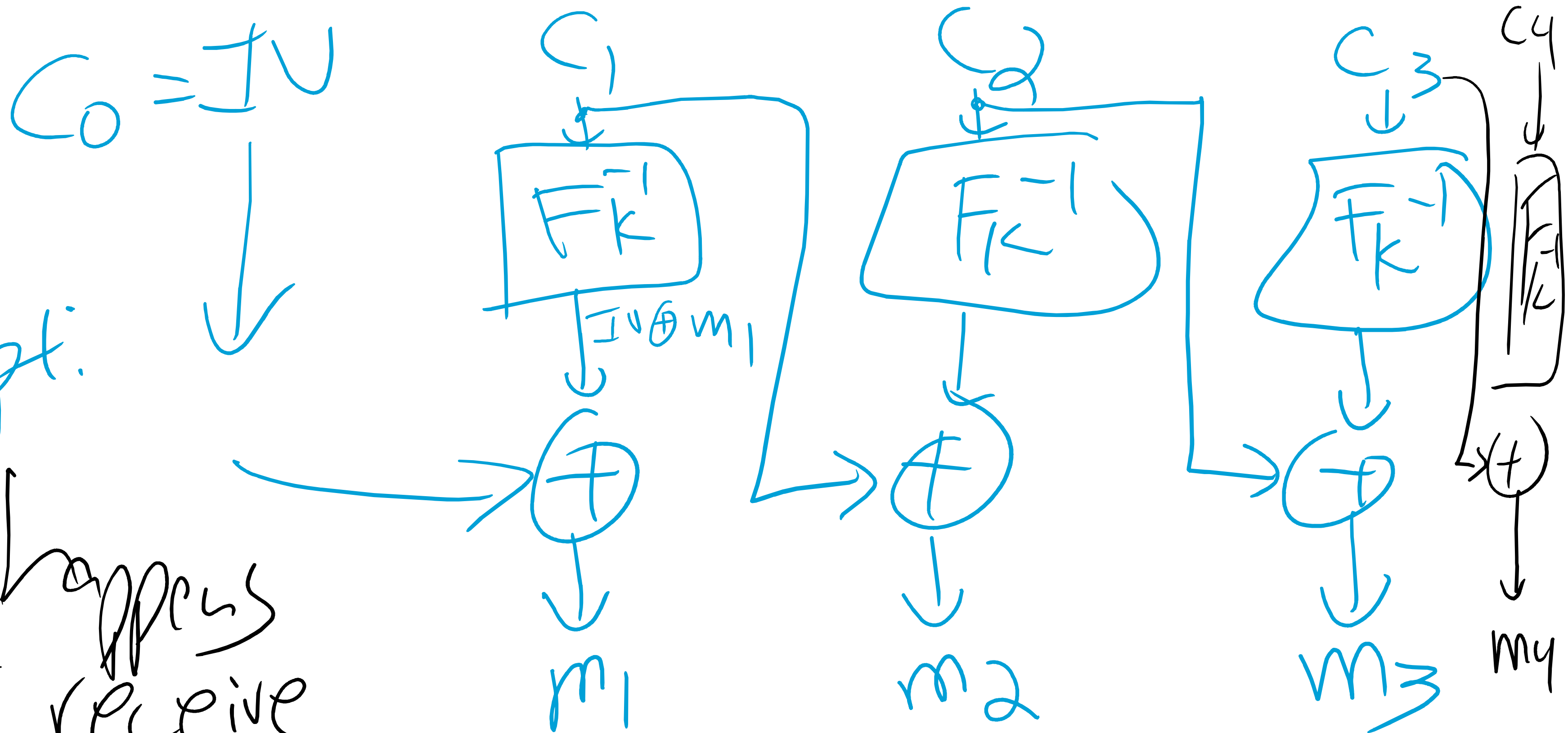
$|C| = 512$

FIGURE 3.7: Cipher Block Chaining (CBC) mode.

Cipher Block Chaining (CBC) mode. To encrypt using this mode, a uniform initialization vector (IV) of length n is first chosen. Then, ciphertext blocks are generated by applying the block cipher to the XOR of the current plaintext block and the previous ciphertext block. That is, set $c_0 := IV$ and then, for $i = 1$ to ℓ, set $c_i := F_k(c_{i-1} \oplus m_i)$. The final ciphertext is $\langle c_0, c_1, \ldots, c_\ell \rangle$. (See Figure 3.7.) Decryption of a ciphertext $c_0, \ldots, c_\ell$ is done

CBC

Decrypt:



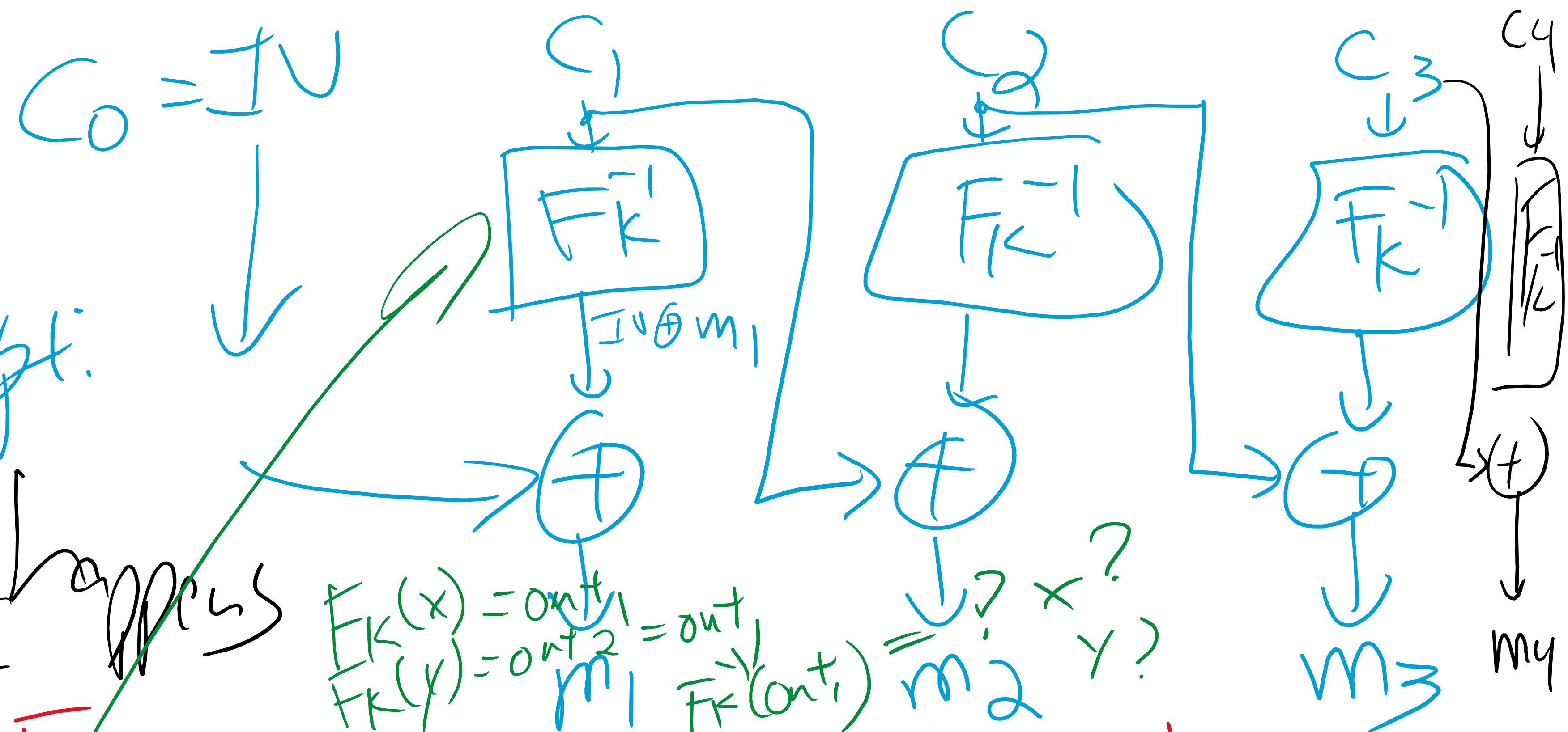
what happens if receive

IV, c_1, c_3, c_4 — m_1 X X m_4

c_2 $\boxed{\text{gone}}$ know c_2 lost

CBC

Decrypt:



what happens if

No because F_k is not a pseudorandom permutation

but is a pseudorandom function

AES is a pseudorandom permutation

Vulnerable to chosen plaintext attack

(i) attacker knows m_1 chosen from $\{m_1^0, m_1^1\}$

(ii) attacker observes IV, c_1, c_2, c_3

(iii) attacker places $IV \oplus m_1^0 \oplus c_3$

(iv) attacker observes c_4, c_5

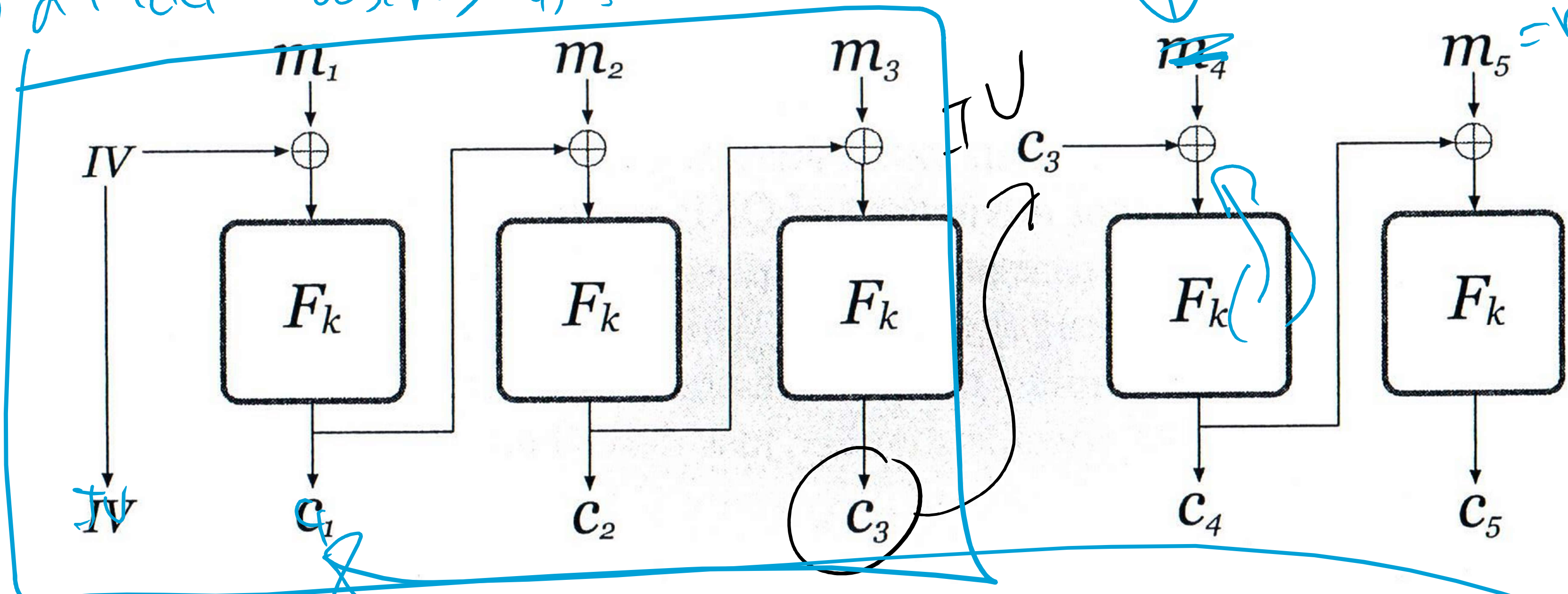


FIGURE 3.8: Chained CBC.

$$c_4 = F_k(c_3 \oplus m_3) = F_k(c_3 \oplus IV \oplus m_1^0 \oplus c_3) = F_k(IV \oplus m_1^0)$$

if $m_1 = m_1^0$ then $c_1 = c_4$

o.w. $m_1 = m_1^1$

$ctr_1 == ctr_2$

$\frac{1}{2^n}$

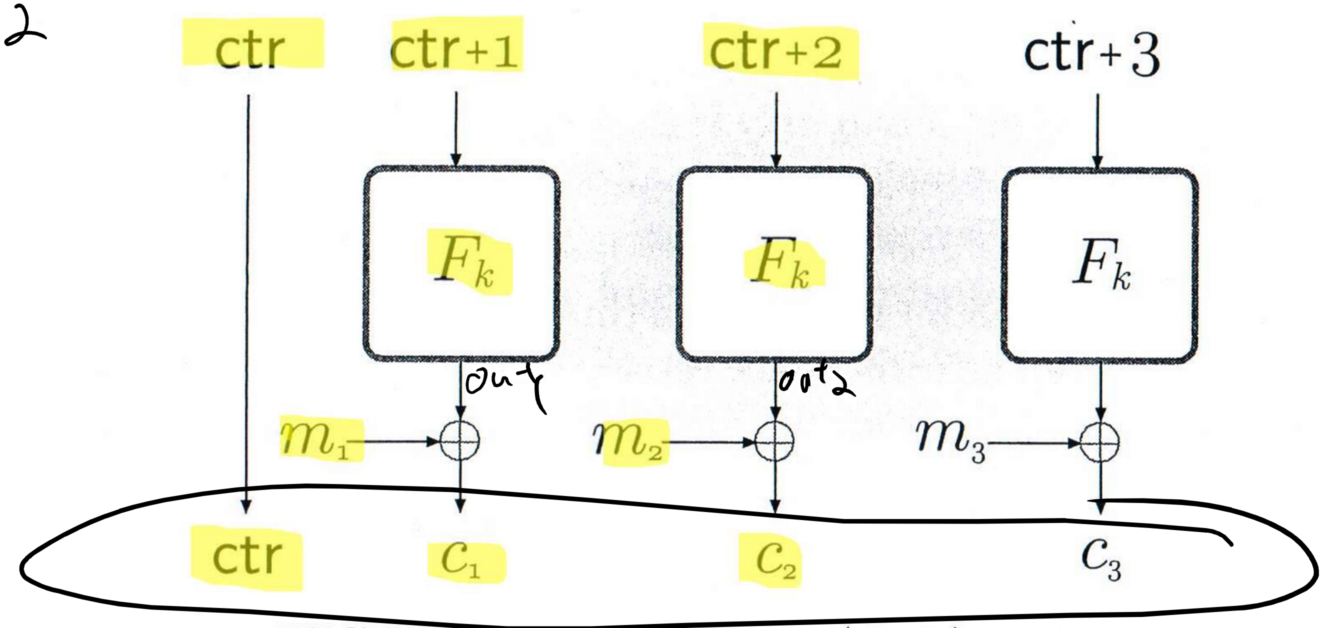
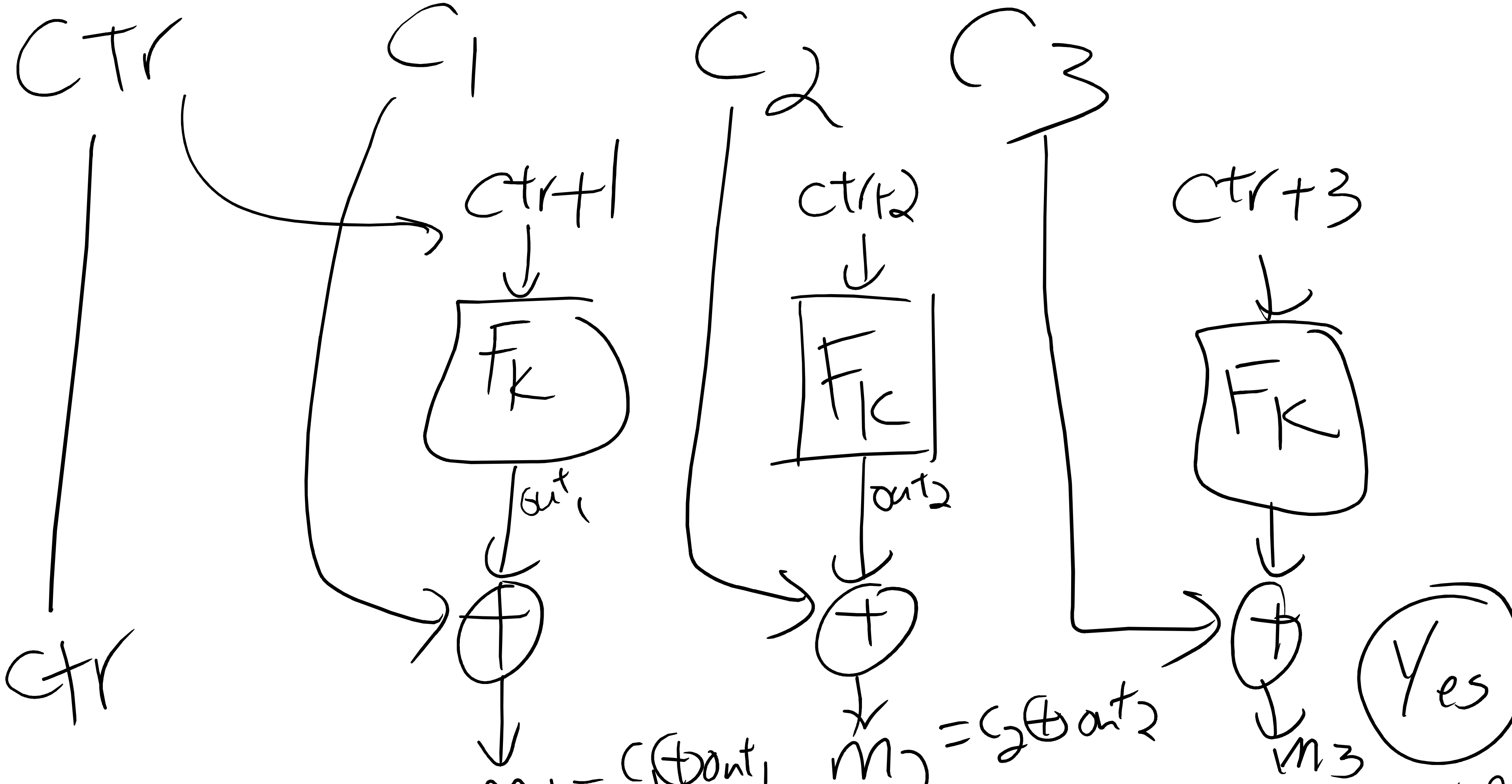


FIGURE 3.10: Counter (CTR) mode.

$c_1 = m_1 \oplus out_1 \Rightarrow m_1 = c_1 \oplus out_1$

Counter (CTR) mode. Counter mode can also be viewed as an unsynchronized stream-cipher mode, where the stream cipher is constructed from the block cipher as in Construction 3.29. We give a self-contained description here. To encrypt using CTR mode, a uniform value $\mathsf{ctr} \in \{0,1\}^n$ is first chosen. Then, a pseudorandom stream is generated by computing $y_i := F_k(\mathsf{ctr} + i)$,



Q: Can we use a pseudorandom function, possible f(x)=f(y), ctr mode?

$m_0^1\ m_0^2 \ldots$

$m_1^1\ m_1^2 \ldots$

Multiple Encryptions

Not covered

No hw

No test

- Ch. 3.4 of Katz and Lindell defines a multiple-message eavesdropping experiment $\mathrm{PrivK}_{A,\pi}^{\mathrm{mult}}$
- Note that this multiple-message experiment $\mathrm{PrivK}_{A,\pi}^{\mathrm{mult}}$ is different than $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}$ defined earlier (indistinguishable encryptions)!
- The end result is that $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}$ is not very useful as a standalone criterion
  - However, $\mathrm{PrivK}_{A,\pi}^{\mathrm{eav}}$ is useful as a building block with formal properties!
- In practice $\mathrm{PrivK}_{A,\pi}^{\mathrm{cpa}}$ is the weakest experiment / definition of interest

Not cover proof, but you should under the result

THEOREM 3.21 *If π is a (stateless)[5] encryption scheme in which* Enc *is a deterministic function of the key and the message, then π cannot have indistinguishable multiple encryptions in the presence of an eavesdropper.*

Is ECB stateless? Yes

$C_0 = AES_k(m_0)$

$C_1 = AES_k(m_1)$

$m_0 == m_1$

$\Rightarrow C_0 == C_1$

[5] Note the ECB is stateless but the rest of the modes presented, including CBC and CTR (and variations w.r.t. the initial vector IV, etc.) are *stateful*.

If an $\boxed{IV}$ is added each time then the IV creates new state each time